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THESIS

THE ANALYSIS OF THE GEOMETRIC FORMULAS FOUND IN CERTAIN FIRST YEAR ALGEBRA TEXTS.

Submitted by

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The Analysis of the Geometric Formulas found in certain First Year Algebra Texts

1. Introduction

The idea that the content of algebra text books was not in agreement with the college board requirements, nor with the modern objectives in algebra, was the thought which prompted this study. In answer to the question on what basis should a text book be selected, we find Maxwell's necessary standards for the selection of a text[1]. Standards are divided into three groups; the first group applied to content and organization includes--"the specific purpose of the book, the purpose as indicated by the statement in the preface, accuracy and reliability of material, freedom from bias and dogmatism, relative values should receive due consideration, scientific studies in education should influence the content, illustrations, and aids in use". Standards of secondary importance are the following: "Reputation of the author and of the publisher, the date of copyright".

1. C. R. Maxwell, The Selection of Textbooks, pp 62-77, 124.

Standards as to the form of the book are given as "general appearance, binding, material of which the book is made, type, and size of the book". In evaluating an algebra text Maxwell tells us that the following points are important: "The point of view, the development, the problem material, and the drill material".

Smith and Reeve's list of criteria in choosing a textbook is the result of the combined judgments of a large number of experienced teachers.[1] Under the heading general considerations, they discuss the purpose of the book, the realization of this purpose, the subject matter, the arrangement of material, and the method of treatment. Some of the sub points considered are as follows: "Does the aim conform to the judgment of scholarly writers of the present time? Does the book emphasize algebra or merely contain the inherited material relating to operations? In the selection of material is a proper balance maintained between the concrete and the abstract?"

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1. David Eugene Smith and William David Reeve, The Teaching of Junior High School Mathematics pp. 204-208.

Other general headings are: "Special Features, including applied problems, reviews, drill, and tests; Aids in Instruction, and Body, Language, And Soul of the Book".

The standards listed here are a fair sample of the literature of the field; they are authoritative, representative, and worthy of consideration, but at the same time subjective and general in scope.

A different technique is followed in this study. Any standards chosen for the selection of a text must be broad in scope if they are to be used in judging an entire book. If, however, one particular point were taken in detail, the standards chosen could be definite and of such a type as to permit objective treatment. The next step is to find, in the objectives of the teaching of algebra, some one point important enough to make this detailed treatment worthwhile and meaningful. Although all the writers do not go as far as Smith and Reeve in saying that the great objective in elementary algebra is the

ability to use formulas,[1] still there is agreement in the fact that the formula is of great importance in first year algebra. Chapter two of this study, which treats of the aims and objectives of algebra, gives more authority for the choice of the formula.

The formulas in first year algebra are of the following types; the geometric mentioned above, business formulas, algebraic formulas, formulas from science, and formulas from everyday life. Because of the wide variety of formulas found in the different texts and the consequent excessive data, it was found necessary to place some limit on the type of formula to be considered. Raleigh Schorling in his book on mathematical objectives,[2] gives the result of a study of seven series of Junior High School Mathematics Texts.

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1. David Eugene Smith and William David Reeve, Objectives in the Teaching of Junior High School Mathematics, p 198.

2. Raleigh Schorling, A Tentative List of Objectives in the Teaching of Junior High School Mathematics, p 90.

In answer to the question what formulas is the pupil expected to know, he concludes from his examination and rating of the twenty one texts that the pupils are expected to know twenty one formulas of which eighteen express geometric relationships. Schorling concludes "When an author tries to explain a formula so that he may dare to hope to have his pupils get some control, he turns to the geometric relations as the most concrete material." Hence the formulas in this study have been confined to the geometric field. Any list of geometric relationships not including proportions seems incomplete, but the author whose objectives are used as an outline in this study, has two separate objectives; one in teaching the formula, another in teaching ratio, proportion, and variation. Since the latter group includes ideas beyond the formula, it was decided to restrict the study to the first objective, the formula, to the exclusion of proportion and variation.

The next question is--what is a formula? The dictionary defintion of a mathematical formula as any general fact, rule,

or principle expressed in algebraic symbols seems to be consistently followed by the texts. Hawkes-Luby-Touton's First Year Algebra say that a formula is a mathematical statement of a law in algebraic symbols.[1] A formula must be a general relation, hence not every statement in algebraic symbols is necessarily a formula. To follow the definition that a formula is an expression of a general fact, rule, or principle, any example such as the following special formula is not included in this study:-'Make a formula for the number of bricks eight inches by four inches required to make a pavement sixty feet long and four feet wide.'

The ten texts used in this study were selected in the following way. Letters were sent to twenty leading publishers of textbooks asking for the name of their most popular first year algebra text. From the replies received, the ten books published from 1925 to 1934 inclusive were selected, taking only one book from each publishing house.

1. Herbert E. Hawkes, William A. Luby, and Frank C. Touton, First-Year Algebra p 253.

II Aims and Objective in Teaching Algebra

1. The Aims as given by Reeve.

In an article in the Mathematics Teacher, Reeve gives a definite list of objectives thought to be desirable for the ninth grade course in algebra.[1] The list which is not exhaustive nor arranged in order of importance was formulated by the author with the cooperation of about two hundred experienced teachers. It is, however, a definite working basis, and because of its agreement with the ideas of so many other writers in the field, it has been used as an outline in comparing the texts in this study. The list includes thirteen algebraic objectives and a list of desirable formulas for use in ninth grade algebra. The Objectives in Teaching the Formulas are as follows:

1. The ability to develop certain rules of mathematics and to translate them into formulas. This means that the pupil should understand the meaning of the formula as a shorthand rule of mathematics. The rule should grow out of his experience if possible. At any rate, he should be

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1. W. D. Reeve, Objectives in the Teaching of Mathematics, pp 385-401.

told what the formula means as far as possible. Here is where algebra begins.

II The ability to translate certain formulas into rules of mathematics. This means that the pupils must know how to use a formula when the need arises. That is, in getting certain required results, he must be taught how to decide which is the proper formula to use.

III The ability to evaluate certain formulas; that is to find the value of certain letters when the values of the others are known. These formulas should be of a difficulty no greater than that found in the operations which the pupils have been taught, or which they may be expected to understand.

IV The ability to derive one formula from another. This means that the pupil must be able to solve a formula for one letter in terms of the other letters in that formula. This involves the ability to solve equations, by means of which solution the subject of the formula is changed. The types of equations involved are as follows:

$2x = 6, \quad x + 5 = 8, \quad x - 4 = 7, \quad \frac{1}{2}x = 9.$

The solution of such equations implies a knowledge of how to use the four fundamental operations.

V The ability to represent by a graph certain formulas of a type no more difficult than F=9c/5 32. This involves the ability to make a table of values for a formula.

VI The ability to understand the idea of the dependence of one quantity upon another. This involves the ability to appreciate the idea of one variable as a function of the other.

The main list of desirable formulas includes seventy, of which thirty nine are geometric.

In the Second Year-book of the National Council of the Teachers of Mathematics there is an article by Smith and Reeve which says,[1] "The great objective in elementary algebra is the ability to use formulas. This means that the pupil

1. David Eugene Smith and William David Reeve. Objectives in the Teaching of Junior High School Mathematics, p 198

should be able to evaluate a formula and to derive one formula from another. The pupil should understand that a formula is a shorthand statement of a rule. He should also see that a simple formula such as he uses may be represented by a graph, realizing that a rule is a translation of a formula, and a graph its picture. The rest of elementary algebra is an elaboration of these principles." Then follows a list of abilities in algebra similar to those given by Reeve, the special abilities to be developed by the formula being the same six with the addition of this seventh--the ability to work with ordinary, simple formulas.

From here on, comparison of definite mathematical aims will be made by referring to Reeve's list of six abilities in the teaching of the formula.

2. The Aims listed by the College Entrance Examination Board.

The College Entrance Examination Board in its requirements for elementary algebra gives the following:[1]

1. College Entrance Examination Board, Definition of the Requirements in Elementary algebra, p^2, p 13.

1. "The meaning, use, evaluation, and necessary transformations of simple formulas involving ideas with which the pupil is familiar, and the derivation of such formulas from rules expressed in words. The following are types of the formulas that may be considered: $V = \frac{4}{3}\pi r^3$ (the sphere), $A = \frac{1}{2} h (b + b')$ (the trapezoid). $S = \frac{1}{2} gt^2$ (falling bodies). $A = p(1+rt)$ (amount at simple interest). $A = p(1+r)^t$ (amount at compound interest). In the work done with formulas, the general idea of the dependence of one variable upon another should be repeatedly emphasized. The illustrations should include formulas from science, mensuration, and the affairs of everyday life. 2. The graph, and graphical representation in general. The construction and interpretation of graphs. The following are types of the material adapted to this purpose: statistical data, formulas involving two variables such as $A = \pi r^2$ and $y = x^2 + 3x - 2$; formulas involving three variables, but considered for the case in which an arbitrary value is assigned

to one of them as $V = \pi r^2 h$ for a fixed value, say 4, of h".

These requirements compare very favorably with the six objectives of Reeve, although it is necessary to go to their second point, graphs, to get Reeve's fifth objective, the graph of a formula. In doing this, graphical representation is included in the list of objectives although statistical graphs are not included in this study. Also, Reeve's fifth objective limits the difficulty of the formula to be graphed to $F=9c/5+32$, a linear function, while the college Board suggests $A=\pi r^2$ and $V=\pi r^2 h$ (for a fixed value of h) which are quadratic functions. On the other points there is agreement.

Both Breslich[1] and Whitcraft assert that progress in mathematical reforms has been retarded by the rigidity of the college entrance examinations, and by the

1. E. R. Breslich, Secondary School Mathematics and the changing curriculum, p328

2. L. H. Whitcraft, Some of the Influences of the Requirements and Examinations of the College Entrance Examination Board on Mathematics in the Secondary School of the United States, p 235

fact that some obsolete material has been retained in the examinations since the 1923 requirements. The latter apparently objects to the definite goals set by the board requirements which have stimulated certain teachers to do better teaching, but which have interfered with the best teaching of another group who would like to do work of a creative nature. Whitcraft also gives twelve topics which have been emphasized because of the examinations, among them are formulas, graphs, and the function concept. He does not agree with the chapter on College Entrance Requirements in the Report of the National Committee where they say,[1] "There is no real conflict between the needs of the students who ultimately go to college and those who do not" because he says "more attention should be given to organizing courses that will meet the needs of the majority of high school students--those who are not going to college".

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1. National Committee on Mathematical Requirements, The Reorganization of Mathematics in Secondary Education, p 66

In April 1933 the College Entrance Examination Board authorized the appointment of a commission on examinations in mathematics, headed by Arnold Dresden, President of the Mathematical Association of America. Among the questions to be taken up was the desirability of revising the present definition of the requirements in Mathematics. The report of this commission, which was submitted to the Committee of Review on April 9, 1935 has been published.[1] The report is not final but makes the following recommendations: "Sharp separation between algebra, geometry, and numerical trigonometry will not be maintained. Understanding of the interrelations between these subjects should be part of the candidates' equipment." "A detailed specification of the topics to be covered by the examination is not desirable, therefore the list indicates the general scope and the point of view from which the material is to be

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1. College Entrance Examination Board, Report of the Commission on Examinations in Mathematics. pp 159-160

considered." The algebra list has three parts; language, ideas, and processes in algebra; functional dependence, and problem solving. Functional dependence includes the following: (a) the formula (e.g., formulas which appear in geometry, in physics, or in other fields within the experience of the candidate), (b) the equation (e.g., linear and quadratic equations in one variable, systems of equations in several variables, their connection with the identity and with the formula), (c) the graph (e.g., representation of statistical data and of relations between two variables given by equations".

Although these recommendations differ from the requirements in effect when this study was started, there are points which may be compared with Reeve's list of objectives. Functional dependence taught by the formula agrees with his sixth point and taught by the graph agrees with his fifth point. As the new examinations begin in 1936 and the old type examinations will be contined through 1938, in comparing the texts later in this study both

requirements will be considered.

3. The Aims as suggested by the National Committee on Mathematical Requirements.

The Report of the National Committee distinguishes three classes of objectives:[1] practical aims--of which one is the ability to understand and interpret correctly graphic representations of statistical data and representations of dependence; disciplinary aims--one of the most fundamental of which is training in functional thinking, that is, thinking in terms of and about relationships; cultural aims which are somewhat less tangible. Functional relation is mentioned again as the one great idea which is best adapted to unify the course. The report also states, "The primary and underlying principle of the course should be the idea of relationship between variables, including the methods of determining and expressing such relationship". There are six general ideas which appear more explicitly in the course, and under the dominance of one

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1. National Committee on Mathematical Requirements, op. cit., pp 6-13.

or another of which all topics should be brought. Among these are the formula and graphic representation. Here we find agreement with the previously mentioned idea of the importance of the formula, the graph, and the idea of dependence.

The specific topics to be taught in algebra, according to the National Committee, include the formula and graphs.[1] "The formula--its construction, meaning, and use as a concise language, as a shorthand rule for computation, as a general solution, as an expression of the dependence of one variable upon another. The work should include translation from English into algebraic language, and vice versa. The nature of the dependence of one variable in a formula upon another should be examined and analyzed, with a view to seeing how the formula works. Graphs and graphic representation in general--their construction and interpretation in representing facts, in representing dependence, in solving problems." These definite algebraic aims contain Reeve's six

1. Ibid., p32

points in the teaching of the formula and only go beyond them in mentioning graphs as representing facts and solving problems, the last two being taken by Reeve in a separate group. The National Committee has two headings: the formula, in which no mention is made of a graph; graphs and graphic representation in general which includes the graph to show dependence. From this comparison we may conclude that Reeve followed the National Committee in selecting his objectives, but did not follow their grouping.

Chapter five of the same report deals with college entrance requirements, and gives the report of an investigation by the National Committee as to the content of high shcool courses most desirable as preparation for college work. There are five topics which are considered most important for this type of work and two of the five are: simple formulas--their meaning and use, the linear and quadratic functions and variation. Attention is called to the agreement between the desires of the college teachers and the National Committee's recommendations. As

the latter includes "functional relationship as the underlying principle of the course" and the "emphasis on the meaning and use of simple formulas," the evident conclusion is that there is no real conflict between the needs of students who ultimately go to college and those who do not.

Hedrick's chapter The Function Concept stresses the fact that the committee recommends not the word function, but that the idea of relationship or dependence between variable quantities be imparted to the pupil.[1] "The teacher should have in mind constantly not any definition to be recited by the pupil, not any automatic response to a given cue, not any memory exercise at all, but rather a determination not to pass any instance in which one quantity is related to another, or in which one quantity is determined by one or more others, without calling attention to the fact, and trying to have the student see how it works."

The function idea in algebra is often brought out by the study of graphs but Hedrick

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1. Ibid., pp. 92-105

says that the very first illustrations given in algebra are essentially functional in character. He believes that from the time letters are first used in place of numbers, questions on the relationships between the letters and questions on the changes in one resulting from changes in the other should be considered. Tables and graphs are other ways of showing dependence and should have their definite place in the course. The student should come to realize that the three concepts, tables, graphs, and algebraic formulas are all representations of the same kind of quantities.

The conclusion on the importance of the function concept, by Hedrick, is one of the strongest arguments in its favor in the earlier literature and reference is repeatedly made to it by later authors. He gives several concrete examples of the many times in life when it would be advantageous to have a mind trained in thinking of the relationships between quantities. Training in such habits of thinking is not only a personal asset but is also a vital element toward the creation of good citizen-

ship. The author questions the transfer of training given by courses that do not emphasize functional relationships, but believes the transfer does operate when mathematics is taught functionally because of the existence of common elements of thought in functional thinking--both in mathematics and in the affairs of life.

Raleigh Schorling[1] tells us, "Teachers are enthusiastic for the program of the National Committee, but they do not always see what things to teach to put the new program into practice," the chief defect of the report being its failure to list specific objectives. He also says that the suggestions given by the National Committee will be recognized for at least a decade, and in discussing Schorling's book later we will see that in formulating his objectives he has taken the committee's recommendations as one of his five criteria.

W. R. Longley mentions two handicaps from which the National Committee suffered,

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1. Raleigh Schorling, op. cit., p. 119

but which were remedied to a certain extent by the college entrance requirements.[1] They are its lack of authority to enfore its recommendations and the fact that its existence was only temporary. In spite of these handicaps little has been written in this particular field without at least a reference to the National Committee.

The effect of the report can be seen, although in varying degrees, by an examination of recent texts and by reading the literature since 1923. Edwin S. Lide says, "In Junior High School, the recommendations of the National Committee on Reorganization, except with respect to the development of functional thinking and emphasis on college preparatory in grade nine, have been carefully followed".[2]

The recommendations of the National Committee, we may conclude, have had a great deal of influence on the recent teaching of algebra, and would have had more influence if specific objectives had been formulated

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1. W. R. Longley, The Function Concept in Algebra, p 6

2. Edwin S. Lide, Mathematics in the Changing Curriculum of the High School, p 491

which would have demonstrated how to teach the function concept.

4. The Aims as given in the survey by Breslich, followed by his own list of objectives.

E. R. Breslich has devoted one chapter of his book on mathematics teaching to aims, and gives a comprehensive survey of the literature of the field.[1] "At present there is no single statement of objectives on which all educators unanimously agree. The formulation of the real objectives is still in a stage of development. The tendency seems to be for each curriculum specialist to set up a system of objectives of his own." Breslich presents the following: Herbert Spencer's list of life activities to attain the ends of education; Franklin K. Bobbit's thought that education should prepare definitely for the activities of adult life; the well known Cardinal Principles of Secondary Education; the practical, disciplinary, and cultural aims of the National Committee on Mathematical Requirements; the practical, disciplinary, and minor functions given by Schultze; Young's list of practical values,

1. Ernest R. Breslich, The Technique of Teaching Secondary School Mathematics, pp 189-199

mathematics as a mode of thought, and other functions of mathematics; and Schorling's four groups of attitudes, concepts, information, and skills or abilities.

Schultze makes no mention of the importance of the formula, but he gives seven reasons for teaching graphs;[1] among them that the student acquires a clear notion of one of the most important notions of advanced mathematics, viz: functionality. This may be compared with Reeve's sixth point on dependence, also with his fifth point on graphs because Schultze goes on to mention the following points in the teaching of graphs: graphic representation of a given numerical table, graphs of physical and geometric formulas as C=44r/7, interpretation of graphs.

Schorling has chosen the objectives of junior high school mathematics on the following five criteria:[2] Objectives Studies, Junior High School Texts, Practice as determined by courses of study, the Outline of

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1. Arthur Schultze, The Teaching of Mathematics, pp 15-29

2. Raleigh Schorling, op. cit., pp3, 74, 106

the National Committee, and a Competent Jury. His list of attitudes, concepts, abilities, and informations includes among the ten abilities, abilities in using a formula. These abilities in using a formula include fourteen formulas to use in computation, of which all but one, the interest formula, are geometric. This computation ability corresponds with Reeve's third point, evaluation. The abilities to translate a rule into a formula and vice versa correspond with Reeve's first and second points, also the ability to solve any of the given formulas for any letter corresponds with Reeve's number four. The subsidiary list includes the evaluation of two more geometric formulas, the ability to write a formula from a table of paired values, the ability to extend a table showing how two related numbers change (included in Reeve's fifth), and the ability to formulate an arithmetic rule from a table. These objectives differ from Reeve in that there is no mention of the graph of a formula, and only slight mention of changing quantities. Breslich criticizes Schorling for not establishing relative values in his study,

also for not including other objectives of the unit on formulas, such as appreciation of the value and usefulness of the formula, and a thorough understanding of the meaning of the formula.[1] Schorling also gives a report from Thorndike's Psychology of algebra as to the result of the examination of forty-four-high school texts to discover what mathematics is needed, in which Thorndike says, "Teachers of many subjects would find it advantageous if they could utilize the mathematical graph--that is, the graphic presentation of a law or a function". He also concludes,"It would be profitable to extend the field of application of the construction of formulas as well as their evaluation, and the presentation of laws by means of mathematical graphs should be encouraged". The following criticism is given by Blackhurst,[2] "The study by Raleigh Schorling is the most thorough and scientific attempt made to date to determine a specific list of objectives for Junior High School Mathematics".

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1. Ernest R. Breslich, The Technique of Teaching Secondary School Mathematics, p 200

2. J. H. Blackhurst, Principles and Methods of Junior High School Mathematics, p82

All writers agree that the definite mathematical objectives should contribute toward the general objectives of education, but as there are different schools of thought for the basic objectives it follows that the definite aims also vary. To quote from Breslich again,[1] "The reader in search of objectives will be confused by the variety of the terminology used. Some of the items listed are clearly general mathematical objectives, others are subject objectives, and most of them are merely important assimilative material which are to contribute to the attainment of the objective". A list of general aims of the teaching of secondary-school mathematics is presented; this list has been compiled from a number of studies relating to objectives, from articles dealing with the subject, and from prefaces of the authors of several text books. The objectives are classified as follows: general educational objectives, general mathe-

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1. Ernest R. Breslich, The Technique of Teaching Secondary School Mathematics pp. 190, 203-20

matical objectives, subject objectives, and unit objectives. Before teaching a unit, the instructor must have a clear understanding of the objectives of mathematics and must decide definitely on the contributions which a unit is to make to these objectives. In his first group of 'powers' we find nineteen algebraic powers, and from that list, the following pertaining to formulas: "The power to understand and use formulas, to solve formulas for a given letter, to evaluate formulas, to translate verbal statements into formulas, to represent formulas graphically". There are also six powers to be derived from the graphical method of representing numerical parts and relationships:"to understand the meaning of graphical representation, to interpret graphs, to make graphs from given data, to represent equations and formulas graphically, to use graphical methods of solving equations, to study the properties of functions graphically."

In this fourth section on aims we have again rather general agreement with Reeve's six objectives. There are points of difference, which have been pointed out, some

of these being differences in emphasis, but on the whole, there are common ideas enough to warrant following Reeve's list as an outline.

5. The aims as given by other writers in the field.

The Seventh Yearbook of the National Council of Teachers of Mathematics entitled The Teaching of Algebra has several articles devoted to functionality among which are the following: Jablonower says,[1] "For elementary purposes we may think of two sense data as being functionality related when a change in one sense datum is invariable accompanied by a change in the other. Algebra is primarily an instrument for expressing such functional relation, and the processes in algebra may be regarded as devices for expressing this functional relation in more than one way, when more than one way is needed. It is the aim of all sciences to discover and express

1. Joseph Jablonower, Recent and Present Tendencies in the Teaching of Algebra in the High Schools, pp13-14

functional relations among their data. Sciences seek to discover how 'it all depends' and to express the exact nature of the dependence". "If algebra is to effect any useful change in attitude of the student toward his environment, social and physical, it is in the contribution that it makes by way of enabling him to see functionality (even though he know not the term function) in that environment. The laws of physical and social sciences are formulations of the functional relations that they have discovered. To know the world is to see it in terms of functional relations."
This quotation strikes the keynote of the later literature. Lennes in the same yearbook says,[1] "For the purpose of secondary mathematics we may say that a function is a quantity which varies in a definite way as some quantity involved in it varies. A function may be represented, or defined, by a verbal statement such as we use in arithmetic, by an equation or formula, by

1. N. J. Lennes, The Function Concept in Elementary Algebra, p55

a table, or by a graph". Right here we have Reeve's first, second and fifth objectives of the formula given as ways of representing a function altho they are parts of a unit on functions, whereas Reeve's sixth point of a unit on formulas is the function or idea of dependence. This is a decided difference in emphasis. Also, there is no mention of Reeve's third point--evaluation. To represent a function by a formula and 'see how it works' some evaluation is necessary. However, this last point is not mentioned because its importance is so slight compared with the resulting idea of dependence. Again, a decided difference in emphasis. Reeve would use a formula to find the area of a rectangle and incidentally discuss changing quantities. Lennes, and the others in his group, casually evaluate a formula when it is necessary, but their chief interest in it is as a representation of a function. Lennes warns against the definition of function at the beginning of the course,[1]

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1. Ibid., p62

saying that practically all the facts that are needed for the study of the changes of a linear function may be brought out by questions.

Breslich in the Seventh Yearbook gives us the following,[1] "Recognition of the dependence of one variable quantity on another related variable is considered by writers to be one of the most important aspects of functional thinking. Thus, the original meaning of the function concept has been broadened. Functional thinking as now taught implies the idea of variable quantity. It is concerned with the relationships which exist between variables, and with the fact that to a value of one corresponds a definite value of the other". The article goes on to emphsize the fact that the idea of dependence should not be isolated as a separate topic but should be distributed throughout the course. Dependence may be represented by verbal statements, tables, graphs, and formulas. This

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1. E. R. Breslich, Measuring the Development of Functional Thinking in Algebra, pp. 94-96.

is the idea given by Reeve in his sixth point--dependence. None of the writers recommend a formal treatment of the function concept, many say the word 'function' should not be used, and all agree on the importance of questions about changes or dependence. If in the formula A=lw, for the area of a rectangle, a fixed value is given to l, then a discussion of the changes in A as w changes will gradually culminate in a real understanding of the formula A=lw.

J. S. Georges speaks of the importance and the desirability of functional thinking as an activity of human life, and of the fact that functional thinking can best be attained through proper mathematical training.[1] His definition is as follows: "Functional thinking is thinking in terms of quantitative relationships, and has its root in the power we have to associate one thing, or idea, with another thing, or idea". Later, in discussing functional thinking, he says that it involves three distinct and at the same time related abilities,

1. J. S. Georges, Functional Thinking as an Objective of Mathematical Education, pp 508-15.

namely: "The ability to recognize mutual dependence between variable and varying quantities, the ability to determine the nature of the dependence or relationship between variable quantities, and the ability to express and interpret the quantitative relationships".

Georges classifies under six main headings, one of which is 'relationships in algebra', the various mathematical concepts, principles, processes, and methods which contribute to the formation of correct habits of functional thinking.[1] He gives nine functional principles to be emphasized in the relationships in algebra, three of which apply to this study. They are as follows: "the algebraic formula, the graph, and areas and perimeters".

Although the writers mentioned in this last group place their emphasis on the teaching of the function concept, they do refer to formulas, tables, and graphs as the means of representing functions. By following Reeve's objectives in examining the texts,

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1. Ibid., pp. 601-08

we will be able to find out how formulas, tables, and graphs are used, whether they are used for purposes of evaluation or as a means of representing a function'.

6. The aims as indicated in the prefaces of the algebra texts.

There is another group of writers whose objectives must be considered; namely, the authors of algebra textbooks. In discussing these aims, mention will be made only of the objectives of teaching of a formula, or of the related objectives, tables, graphs, and functions.

Book One on our list claims to have incorporated the recommendations of the National Committee in spirit as well as in letter. In this book there is a thorough investigation of a small number of formulas which the pupil can really understand and use to good advantage.

Book Two does not mention the National Committee but claims to conform with the latest revision of the College Entrance Examination Board requirements. There is increased use of the formula. "Almost every topic in arithmetic is stated, reviewed,

and further developed by aid of the formula, and simple formulas are taken from various occupational fields. The devising of formulas at sight by the pupil, and the translation of others into words, develops the algebraic sense better perhaps than any other drill." Graphs are used systematically.

Book Three says its treatment of graphs is unique--functional, algebraic graphs are inserted in the text in connection with the topic they exemplify. As to the formula, there is an introductory chapter on algebraic notation and the formula which leads through a treatment of the formula to types of simple equations arithmetically solved.

The preface of Book Four does not mention the formula although it says the book meets the requirements of the College Board, meets all the recommendations of the National Committee, and its methods of presenting the various topics conform with the best modern practices.

Book Five gives seven immediate objectives, three of which are formulas, functional relations, and graphs. "Formu-

-las are the central topic of Chapters I and II and are featured throughout the text. The meaning of a formula, the various ways of securing a formula, the three uses of a formula and a desirable form for the solution of a problem by a formula are emphasized as essential to mastery of this tool." The three uses as explained in the text are the following: evaluation for the subject, evaluation for another letter, and to show changing quantities. Here we have Reeve's first, third, fourth and sixth objectives. The preface also states that functional relationship is introduced in an informal manner early in the course, is referred to repeatedly, and is finally drawn together and formalized in Chapter Fifteen.

Book Six has made the work on graphs an integral part of the course by placing it in or adjacent to related material. Work on formulas which was a feature of a previous edition, receives even more careful and full treatment than before. "Varied practice is given in the evaluation of formulas. Such work in deriving formulas as is within the range of the student's ex-

perience is given." We can find here Reeve's first and third objectives.

In Book Seven there is no mention of the formula although one of the outstanding features of the book is the new and interesting treatment of functions and graphs. The material in this book is also based upon the conclusions of the National Committee and the College Entrance Examination Board.

Careful consideration has been given to the following in selecting the material for Book Eight: the most modern courses of study including the Report of the National Committee and the requirements of the College Entrance Examinations. There are four practical applications of algebra emphasized; among them are the formula and the graph, which are discussed with special stress on the dependence of variables.

The authors of Book Nine believe the text is in accordance with the spirit as well as the literal recommendations of the National Committee. The special features of the textbook are the following: "Its emphasis upon the formula--its construction, meaning, and use as a concise language, as a

shorthand rule for computation, as a general solution of a problem, and as a way to represent the dependence of one quantity upon another". This is almost a direct quotation from the aims of the National Committee. Two other special features concern the treatment of graphs as a method of representing the relationship between numbers that change together, and the attention given to teaching the concept of dependence.

The material in Book Ten was selected in accordance with the following belief of the author: "The underlying purpose of a course in elementary algebra is to develop the power to represent quantitative relationships by formulas and equations; to develop the power to interpret such expressions of relationship; and to develop the skills needed in the computation involved in using formulas and equations". Three of Reeve's objectives are contained in this author's belief. The preface goes on to say that all teachers realize the difficulty to the pupil of stating a formula or of writing an equation, hence the book gives adequate help in this section of the

work. The stating of the equation for a problem and the writing of a formula seem to be synonymous in the author's mind. We are concerned in this study, not with the formula for a special case or the equation for a praticular problem, but with general geometric formulas.

Three of the ten books, Four, Seven, and Eight state that they are following the recommendations of the National Committee and of the College Entrance Examination Board requirements; two of them, One and Nine, mention only the National Committee; and one, Book Two, mentions only the College Board. That is, five of the ten books whose prefaces were examined are said to be based on the ideas of either the College Board or of the National Committee. Books Four and Seven, two of the three books which mention both authorities, do not have the word formula in the prefaces; but their reference to these authorities indicates their consideration of this aim. Book Three has nothing comparable with Reeve's objectives of the formula or those of any other writer previously discussed in this chapter. Books

One and Two speak merely of the meaning of a formula which may be considered as Reeve's first and second objectives. Books Seven and Eight speak merely of dependence, Reeve's sixth point, while Book Six speaks only of the third point, evaluation. Book Ten includes Reeve's first three points, Book Nine takes in the first three and the sixth, while Book Five includes all but the fifth point.

III Examination of the Algebra Texts.

The ten algebra texts whose selection was described in the introduction have been arranged in alphabetical order, and reference will be made to them by number. The texts were examined carefully and any geometric formulas found were noted. As mentioned before, the thirty-nine geometric formulas in Reeve's list of Desirable Formulas for Ninth Grade Algebra form the basic list, and the formulas found in the algebra texts and not mentioned by Reeve have been added.

1. Following is the complete list of formulas.

1.	$A = lw$	Area of a rectangle
2.	$A = bh$	Area of a parallelogram
3.	$A = \frac{1}{2} bh$	Area of a triangle
4.	$A = h/2 (b+b')$	Area of a trapezoid
5.	$P = 2 (l+w)$	Perimeter of a rectangle
6.	$P = 4 s$	Perimeter of a square
7.	$P = a+b+c$	Perimeter of a triangle
8.	$P = 3s$	Perimeter of an equilateral triangle
9.	$C = 2\pi r$	Circumference of a circle
10.	$C = \pi d$	Circumference of a circle
11.	$A = \pi r^2$	Area of a circle
12.	$A = \pi d^2/4$	Area of a circle

13.	$A = \frac{1}{2} ap$	Area of a regular polygon
14.	$S = 2\ lw + 2\ lh + 2hw$	Total area of a rectangular solid
15.	$S = 6\ e^2$	Total area of a cube
16.	$L = 2\pi r\ h$	Lateral area of a circular cylinder
17.	$S = 2\pi rh + 2\pi r^2$	Total area of a circular cylinder
18.	$L = \pi rl$	Lateral area of a circular cone
19.	$L = Cl/2$	Lateral area of a circular cone
20.	$S = \pi rl + \pi r^2$	Total area of a circular cone
21.	$S = 4\pi r^2$	Area of a sphere
22.	$S = \pi d^2$	Area of a sphere
23.	$V = lwh$	Volume of a rectangular solid
24.	$V = e^3$	Volume of a cube
25.	$V = \pi r^2 h$	Volume of a circular cylinder
26.	$V = Bh$	Volume of a circular cylinder
27.	$V = 4/3\pi r^3$	Volume of a sphere
28.	$V = 1/6\pi d^3$	Volume of a sphere
29.	$A = \pi(R^2 - r^2)$	Area of a ring
30.	$c^2 = a^2 + b^2$	Pythagorean relation
31.	$c = 360/n$	Central angle of a circle
32.	$a + b + c = 180$	Sum of the angles of a triangle
33.	$d = \sqrt{a^2 + b^2 + c^2}$	Diagonal of a cube
34.	$A = n\pi r^2/360$	Area of a sector of a circle
35.	$l = n\pi d/360$	Length of an arc of a circle
36.	$h = a/2\sqrt{3}$	Altitude of an equilateral triangle

37.	$A = a^2/4\sqrt{3}$	Area of an equilateral triangle
38.	$d = a\sqrt{2}$	Diagonal of a square
39.	$E = 1.1547R$	Cube inscribed in a sphere
40.	$S = \pi r$	Length of a semicircle
41.	$C = 2\sqrt{\pi A}$	Circumference of a circle in terms of the area.
42.	$s = ac/360$	Length of an arc.
43.	$P = 2s+b$	Perimeter of an isosceles triangle
44.	$P = 2c+a+b$	Perimeter of an isosceles trapezoid
45.	$P = ns$	Perimeter of a regular polygon
46.	$S = 4(l+w+h)$	Sum of the lengths of the edges of a rectangular solid
47.	$S = 12e$	Sum of the lengths of the edges of a cube
48.	$4m^2 = 2(a^2+b^2)-c^2$	Median of a triangle
49.	$D = n/2\ (n-3)$	Number of diagonals of a polygon having n sides
50.	$P = 5c$	Perimeter of a pentagon
51.	$A = c^2/4\pi$	Area of a circle
52.	$A = \frac{1}{2}\ d^2$	Area of a rhombus
53.	$K = \sqrt{s(s-a)\ (s-b)\ (s-c)}$	Area of a triangle
54.	$A = 2h\ (l+w)+lw$	Lateral area of the sides and one base of a rectangular solid
55.	$A = 2l\ (r+s)+r^2+s^2$	Area of the frustum of a square pyramid
56.	$S = \frac{1}{2}\ pl$	Surface of a pyramid
57.	$S = 2e^2+4\ eh$	Surface of a prism with a square base

58.	$L = \pi s r_1 + \pi s r_2$	Area of the curved surface of the frustum of a cone
59.	$A = b\sqrt{d^2 - b^2}$	Area of a rectangle in terms of its diagonal and base
60.	$A = \pi (R+2r)(R-2r)$	Area left when four circular holes are cut from a large circular plate
61.	$A = s^2$	Area of a square
62.	$A = (36\pi V^2)^{1/3}$	Area of the surface of a sphere
63.	$A = \pi r E/180$	Area of a triangle on a sphere
64.	$A = .7854\ d^2$	Area of a circle
65.	$V = \pi R^2 h - \pi r^2 h$	Volume of a hollow cylinder
66.	$V = 1/3 \pi r^2 h$	Volume of a cone
67.	$V = 1/3 \pi h\ (l^2 - h^2)$	Volume of a cone
68.	$V = 1/3\ as^2$	Volume of a square pyramid
69.	$V = \pi r^2/2\ (h+h')$	
70.	$V = Bh$	Volume of a prism
71.	$V = h/6\ (a+b+4m)$	Volume of a prismatoid
72.	$V = 1/3\ Bh$	Volume of a pyramid
73.	$V = 1/4 \pi d^2 h$	Volume of a circular cylinder
74.	$V = 1/3\ lwh$	Volume of a pyramid
75.	$V = s^2 h$	Volume of a prism with a square base
76.	$V = \frac{1}{2} Bh$	
77.	$V = 1/3 \pi h\ (r^2 + r_1^2 + rr_1)$	Volume of the frustum of a cone
78.	$V = h/3\ (b_1 + b_2 + \sqrt{b_1 b_2})$	Volume of the frustum of a pyramid
79.	$V = A^{\frac{3}{2}}/6\pi$	Volume of a sphere

80.	$V = 1/3\ Bh$	Volume of a cone
81.	$A_1 + A_2 = 180$	Two supplementary angles
82.	$x + y = 90$	Two complementary angles
83.	$A = (n-2)\ 180$	Sum of the angles in any polygon
84.	$A = {}^{n-2}/n'\ 180$	Each angle of an equiangular polygon

2. The total number of different formulas found in each book varies from twenty-one in Book Six to forty-two in Book Ten as shown in the following list.

Table 1: The total number of Formulas found in each text.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total No. of Formulas	32	22	38	31	34	21	22	39	25	42

Only six of the eighty-four formulas are found in all ten books. They are: formula #3 (Area of a triangle, #4 (Area of a trapezoid), #11 (Area of a circle), #25 (Volume of a circular cylinder), #27 (Volume of a sphere), and #30 (The Pythagorean relation). Six more are common to nine of the books. They are the following formulas: #1 (Area of a rectangle), #5 (Perimeter of a rectangle), #6 (Perimeter of a square), #9 (Circumference of a circle), #23 (Volume of a rectangular solid), and #61 (Area of a square). All but this last one are numbered less than forty and are therefore on Reeve's list. The three formulas found in eight books are: #17 (Total area of a circular cylinder), #21 (Area of a sphere), and #66 (Volume of a cone). Seven formulas are used in seven of the algebra texts. They are: #2 (Area

of a parallelogram), #10 (Circumference of a circle), #16 (Lateral area of a circular cylinder), #24 (Volume of a cube), #32 (Sum of the angles of a triangle, #70 (Volume of a prism), and #72 (Volume of a pyramid). Six of the books contain four formulas: #7 (Perimeter of a triangle), #29 (Area of a ring), #36 (Altitude of an equilateral triangle), and #38 (Diagonal of a square). That is, twenty-six of the eighty-four formulas are found in six or more of the algebra texts. Five of the formulas from Reeve's list were not to be found in any of the ten texts. They are: #22 (Area of a sphere) altho it is found in a different form as #21 in eight books, #31 (Central angle), #34 (Area of a sector), #35 (Length of an arc) which in its different form (#42) is found in two of the books, and #39 (Cube inscribed in a sphere). Thirty-three of the formulas have found their place on the list because of their presence in only one book. Of the thirty-three, seven are from Book Ten, six are from Book Eight, and five are from Book Three. The following table shows this distribution of the formulas through the ten books.

Table 2: The Number of Formulas common to the Various Books together with the Specific Numbers of the Formulas Found in the Texts.

The table reads: Of the eighty-four formulas, six are found in all ten books. They are #3, 4, 11, 25, 27, 30 (as numbered on pages 42-46)

Total Number of books	10	9	8	7	6	5	4	3	2	1	0
Total Number of formulas found in the different books.	6	6	3	7	4	4	3	4	9	33	5
The specific numbers of the formulas found in the different books. (For reference see List of formulas, pages 42-46).	#3 4 11 25 27 30	#1 5 6 9 23 61	#17 21 66	#2 10 16 24 32 70 72	#7 29 36 38	#14 15 37 54	#8 28 53	#20 26 71 80	#12 33 42 43 47 56 81 82 84	#13 18 19 40 41 44 45 46 48 49 50 51 52 55 57 58 59 60 62 63 64 65 67 68 69 73 74 75 76 77 78 79 83	#22 31 34 35 39

The question--what formulas is the pupil expected to know--is answered by Schorling after an inventory of the content of seven series of Junior High School Mathematics Texts.[1] He set down these two criteria: the formula must appear in four of the seven series concerned, there must be some slight hint or clue given to the pupil as to what the formula is about, together with practice and later recall. Schorling found twenty-one formulas of which all but five can be found in the preceding chart as being in six or more of the books examined.

Book Four gives a list of fourteen formulas to be known and understood. The following chart compares the formulas found in the upper half of our list, that is in six or more of the texts with the results of Schorling's study and with the list given in Book Four.

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1. Raleigh Schorling, A Tentative List of Objectives for the Teaching of Junior High School Mathematics p 87

Table 3: Comparison of the Most Widely Used Formulas as Found in this Study with Those Found in Schorling's Inventory of the Content of Seven Series of Junior High School Mathematics Texts and with Those Listed in Book Four.

The Numbers of the Formulas Found in Six or more Books	The Formulas Found in Schorling's study which correspond with Column I	The Formulas Given in list in Book Four which correspond with Column I
#3	✓	✓
4	✓	
11	✓	✓
25	✓	✓
27	✓	
30		
1	✓	✓
5		
6		
9	✓	
23	✓	✓
61	✓	✓
17		✓
21	✓	
66	✓	
2	✓	✓
10	✓	✓
16	✓	✓
24		✓
32		
70	✓	
72	✓	
7		
29		
36		
38		
	Also #26, #18, interest, percentage, distance.	Also #15, interest, distance.

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From the above chart we can see some agreement in the ten books examined. From Schorling's list of twenty-one formulas, of which eighteen are geometric, sixteen are found in six or more of our texts. Book Four's list of fourteen contains twelve geometric formulas, eleven of which are found in six or more books, and the twelfth one is found in five of the books.

3. In discovering how many formulas aid in reaching Reeve's first objective: Ability to develop certain rules of mathematics and to translate them into formulas; the following subdivisions have been made. Slight explanation, Detailed explanation,

Pupil writes formula from a given rule (originally),
" " " " " " " (as review),
Pupil writes formula from previous knowledge,
Pupil derives formula from figure and explanation,
Pupil derives fromula from another,
Rule recalled and formula given,
Pupil writes formula (as review).

The range of formulas given with slight explanation is from one formula in Books Five and Eight to seventeen in Book Three.

Table 4: The Number of Formulas Given with only Slight Explanation in the Various Texts.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas Slight Explanation	4	5	17	5	1	11	9	1	3	12

Detailed explanations are given for only two of the formulas in Book Ten but for fifteen of the ones in Book Four.

Table 5: The Number of Formulas Given with Detailed Explanation in the Various Texts.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas Detailed Explanation	5	3	5	15	7	4	6	4	3	2

In some cases, a rule is stated and the pupil asked to write the formula. This is sometimes the manner of introducing the formula: Book Five does this eighteen times while Books Two and Six do not use that method at all. As a means of review, Book Three follows this system eight times, while Books Two, Four, Six and Seven do not.

Table 6: The Number of Formulas in the Texts Written from a given Rule.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas to write from given rule										
Originally	8	0	8	2	18	0	1	11	2	1
As Review	3	0	8	0	2	0	0	1	1	2

The pupil may be instructed merely to write

the formula for the area of a square. Book Nine employs this method nine times and Book Seven only once.

Table 7: The Number of Formulas to be Derived by the Pupil from his Previous Knowledge.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas Derived from Previous Knowledge	4	6	3	3	3	3	0	3	9	2

A formula may be introduced by showing the figure, giving some explanation, and then telling the pupil to write the formula. Book Eight uses this system six times while Books One and Three use it not at all.

Table 8: The Number of Formulas Derived from a Figure and from an Explanation

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas Derived from Figure and Explanation	0	3	0	3	3	1	1	6	3	5

A pupil may be asked to derive one formula from another; for example, #41 (Circumference of a circle in terms of its area) to be derived from #9 (Circumference) and #11 (Area of a circle). Books Three and Nine have three formulas worked out in this way, while Books Two, Four, and Five have none.

Table 9: The Number of Formulas which are Derived from other Formulas

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas Derived from other Formulas	1	0	3	0	0	2	2	1	3	2

Book Eight repeats an arithmetic rule and then gives the formula eleven times, while this is not done in Books Two, Four, and Five.

Table 10: The Number of Formulas given Following the Repetition of the Arithmetic Rule.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas Given after Rule is Repeated from Arithmetic	1	0	1	0	0	4	5	11	3	6

Review lessons often contain a statement such as "Write the formula for the perimeter of a rectangle." Book Three has thirty-three such sentences, but Books Six and Seven have none.

Table 11: The Number of Formulas to be Written as a Means of Review.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas Written as Review	11	12	33	32	14	0	0	20	17	16

The summary of tables four to eleven may be found on page one hundred and three, table 66.

The ability to translate certain formulas into rules of mathematics, Reeve's second objective has been subdivided as follows: Formula given, pupil writes translation into mathematics originally, or as review; Formula given then rule given; and finally directions when to use certain formulas. Book One uses this translation into mathematics most often, having thir-

ty original translations and twenty-five review translations as contrasted with no original translations in Books Four, Six, and Seven, and no review translations in Books Six and Seven.

Table 12: The Number of Formulas which the Pupil Translate into Rules.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Formula given: pupil writes translation										
Originally	30	9	7	0	2	0	0	6	5	3
As review	25	11	15	5	1	0	0	3	3	1

In only one of the books, Number Two, is the second idea followed; formula given first, rule following.

Table 13: The Number of Formulas for which the translation is Given Following the Formula

The Books by Number	1	2	3	4	5	6	7	8	9	10
Formula given: rule given	0	1	0	0	0	0	0	0	0	0

Directions when to use certain formulas are given in only one book, Number Nine, although Reeve gives this point as part of the meaning in his second objective

Table 14: The Number of Times Directions for the Use of Formulas are Given.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Directions when to use certain formulas	0	0	0	0	0	0	0	0	11	0

Note must be made of the fact that neither Book Six nor Book Seven has a single example of the transla-

tion of a formula into a rule. The summary of tables twelve, thirteen and fourteen may be found on page one hundred and four, table 66.

Sinceall the writers agree that the pupil should be taught the meaning of a formula and since Reeve's first two objectives, translation of rules into formulas and formulas into rules best exemplify this idea; the number of different formulas which aid in reaching these two objectives are listed together. Although Book Ten has the greatest number of formulas (forty-two), only thirty of them are included in points I and II, whereas Both Book Three and Book Eight have thirty-six of their formulas explained. Book Six with the smallest number of formulas (twenty-one), presents only three of them without any clue as to their meaning. Both Book One and Book Seven have all their formulas explained. An unexplained formula may be presented for evaluation: as, if $s = 5$, and $h = 10$, find V in the formula $V = s^2h$; or for transformation: solve the formula $V = s^2h$ for s. If there is some suggestion as to the meaning of the terms, the formula find its place under one of the subheadings of I or II. Three of the texts use formula 17 (Total area of a circular cylinder) for the purpose of evaluation, and give no hint as to its meaning. Three of the texts use formula 71 (Volume of a prismatoid); one of them for evalua-

tion, the other two for transformation, and no explanation whatsoever is given.

Table 15: The Following Table Shows the Number of Explained and Unexplained Formulas Found in the Ten Texts.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas	32	22	38	31	34	21	22	39	25	42
Number of Different Formulas Explained	32	18	36	28	32	18	22	36	19	30
Number of Unexplained Formulas	0	4	2	3	2	3	0	3	6	12

The specific numbers of the unexplained formulas are listed on page one hundred and two, table 65.

The evaluation of a formula, that is, finding the value of certain letters when the values of certain others are known--conceded to be the practical reason for teaching a formula--includes the following points. A formula may be evaluated for its subject, as, find the value of A in the formula A = lw, when l and w are known; a formula may be evaluated for another letter, as, find l in the formula A = lw when A and w are known; or a formula may be evaluated for another letter after its 'subject has been changed', as, in the formula l = A/w find l when A and w are known. Taking the first point, the range of evaluations is from one hundred seventy-one in Book Three to only thirty one in Book Seven. Evaluating for another letter: only once in

Book Nine and forty-nine times in Book One; evaluating after changing the subject is required thirty-eight times in Book Three and not at all in Book One. In counting these evaluations, if the same formula was used six times for computation in one exercise, this was counted as six evaluations in that exercise. Book Six on page 252 says "The workmanlike way of solving all such problems" (evaluating for a letter other than the subject) "is first to solve the given formula for the required unknown and then to substitute the given numerical values in the result so obtained." Hence, from there on, any such problem was considered first as transformation, then evaluation after the subject was changed. The range of the total number of evaluations is very great, fifty-seven in Book Nine to two hundred and twenty-seven in Book Three.

Table 16: The Number of Times Formulas are Evaluated; Including Evaluation for the Given Subject, for Another Letter, after the Change of Subject, and the Total Number of Evaluations.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Evaluations for Given Subject	131	65	171	135	54	51	31	119	43	64
Number of Evaluations for Another Letter	49	10	18	21	42	2	25	16	1	15
Number of Evaluations after change of Subject	0	15	38	3	5	23	11	10	13	11
Total Number of Evaluations	180	90	227	159	101	76	67	145	57	90

The number of different formulas to be evaluated is much more uniform than the number of evaluations. Book Three in its two hundred and twenty-seven evaluations uses only thirty-four different formulas, while Book One has thirty different formulas and one hundred eighty evaluations. This is consistent with the statement in Book One, "Formulas are thoroughly mastered by means of a thorough investigation of a small number of formulas which the pupil can really understand."

Table 17: The Number of Different Formulas which are Evaluated in the Texts.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Different Formulas Evaluated	30	18	34	26	24	14	16	34	18	27

The ability to derive one formula from another, to solve a formula for any letter, to change the subject of a formula, or the necessary transformation of a formula is Reeve's fourth objective. He gives the following types of equations: $2x = 6$, $x+5 = 8$, $x-4 = 7$, and $\frac{1}{2}x = 9$, to which has been added a fifth type, equations involving radicals. This addition was made because of the many times a formula such as #11 ($A = \pi r^2$) must be solved for r. $A = lw$ is an example of the formulas of the first type of which Book Nine has twenty-six transformations and Books Six and Seven have none.

Table 18: The Number of Transformations of the Type $2x = 6$

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Transformations of Type $2x = 6$	3	15	9	19	6	0	0	4	26	17

The second type, exemplified by #7 ($P = a+b+c$), is transformed only once in Books Two and Nine and thirteen times in Book Four.

Table 19: The Number of Transformations of the Type $x+5 = 8$

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Transformations Type $x+5 = 8$	4	1	4	13	4	5	3	3	1	4

Formula #29 ($A = \pi(R^2-r^2)$ is one of the few which may be cited as an example of the third type, of which there are no transformations in any of the books.

Table 20: The Number of Transformations of the Type $x-4 = 7$

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Transformations Type $x-4 = 7$	0	0	0	0	0	0	0	0	0	0

Type four, of which formula #3 ($A = \frac{1}{2}bh$) is a good example, has its subject changed twenty-three times in Book Four and zero times in Book Seven.

Table 21: The Number of Tranformations of the Type $\frac{1}{2}x = 9$

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Transformations Type $\frac{1}{2}x = 9$	10	3	6	23	10	2	0	5	5	9

The radical or fifth type, for which we may give formula #11 ($A = \pi r^2$) as an example if it is solved for r, is not required in Book Seven but is used twelve times in Book Six.

Table 22: The Number of Transformations of the Type $A = \pi r^2$

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Transformations Type $A = \pi r^2$	4	7	10	7	4	12	0	5	1	8

The totals of these five charts show that Book Four changes the subject of a geometric formula sixty-two times and Book Seven only three times.

Table 23: The Total Number of Transformations

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Transformations	21	26	29	62	24	19	3	17	33	40

It can easily be seen from this chart that Book Seven's three transformations is an extreme case, as the next lowest number is seventeen for Book Eight. The Books vary again, but not to such an extent, in the number of different formulas which are used for transformation. Book Seven has its three transformations of three different formulas, while Book Four which had the greatest number of transformations (sixty-two) uses only sixteen different formulas.

Table 24: The Total Number of Transformations Compared with the Number of Different Formulas Transformed.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Transformations	21	26	29	62	24	19	3	17	33	40
Number of Different Formulas Transformed	9	13	18	16	13	10	3	10	13	24

The summary of tables eighteen to twenty-four is in Table Sixty-six on page one hundred and four, table 66.

Reeve includes in his fifth point on drawing graphs, the ability to make a table of values for a formula, hence the subdivisions on point five are as follows: tables of values to be made for a formula, graphs drawn from these tables, graphs drawn without mentioning the construction of a table, and the number of times values are read from a graph. A formula may be evaluated by reading points from its graph; a problem of this type is classed here not as an evaluation, but as a reading from the graph. Six of the books examined, namely, One, Two, Four, Six, Seven, and Ten, have no tables to be made for geometric formulas, while Book Three has twelve.

Table 25: The Number of Times Tables are made for Geometric Formulas

The Books by Number	1	2	3	4	5	6	7	8	9	10
Tables of Values	0	0	12	0	6	0	0	4	1	0

Book Nine, which has one table to be constructed,

does not complete the picture by drawing the graph, so there are seven books which have no graphs drawn from tables of values.

Table 26: The Number of Graphs Drawn from Tables

The Books by Number	1	2	3	4	5	6	7	8	9	10
Graphs Drawn from Tables	0	0	6	0	1	0	0	4	0	0

Some of the books require the graph of a formula but do not mention making a table to help in its construction. Book Two follows this idea ten times and Book Eight not at all, the latter having its four graphs drawn from previously constructed tables.

Table 27: The Number of Graphs Drawn without Tables

The Books by Number	1	2	3	4	5	6	7	8	9	10
Graphs Drawn (No Tables)	1	10	5	4	1	4	2	0	4	3

As some of the texts follow both methods we shall consider the total number of graphs to be drawn; the range being from one in Book One to eleven in Book Three.

Table 28: The Total Number of Graphs Drawn

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Graphs Drawn	1	10	11	4	2	4	2	4	4	3

All of the books test the pupil's ability to read values from a graph by questions such as the following, "From the graph of the formula $A = \pi r^2$, find the value of A when $r = 3$." Sometimes these values are to be

read from a graph given in the book, and sometimes from a graph which the pupil is to construct, but the readings in both cases have been grouped. Books Seven and Ten have only two of these readings while Book Five has twenty-five, and only two graphs to be constructed.

Table 29: The Number of Readings from Graphs

The Books by Number	1	2	3	4	5	6	7	8	9	10
Readings from Graphs	7	12	18	16	25	11	2	16	19	2

The different formulas found in this fifth point show the usual differences: Book One has one formula, one graph, no tables, and seven readings from the graph; Book Seven has two formulas, two graphs, two readings, and no tables; Book Three uses ten formulas, requires twelve tables, eleven graphs and eighteen readings, this last book having the greatest number of formulas, tables, and graphs and ranking third on the readings. The compete picture is as follows:

Table 30: A Summary of Tables 25, 28, and 29 compared with the Number of Different Formulas Used

The Books by Number	1	2	3	4	5	6	7	8	9	10
No. of Tables	0	0	12	0	6	0	0	4	1	0
No. of Graphs	1	10	11	4	2	4	2	4	4	3
No. of Readings	7	12	18	16	25	11	2	16	19	2
No. of Different Formulas	1	5	10	4	7	3	2	4	5	4

When we come to Reeve's sixth point-the ability to understand the idea of the dependence of one quantity upon another, which involves the ability to appreciate the idea of one variable as a function of the other--we reach a point where objective treatment seems a little difficult. We can recall from the chapter on aims and objectives such bits as 'the spirit of the function', 'training in functional thinking', 'the idea of changing quantities', 'thinking in terms of quantitative relationships', 'how it all depends', 'how the function is marching', 'not the definition of the word function', and 'not a formal treatment of the function concept'. We can also recall the following: "The understanding of the way a change in one variable affects others related to it is an important characteristic of the functional thinking."[1] "Practically all the facts that are needed for the study of the changes of a linear function may be brought out by questions."[2] This statement from Lennes is limited to linear functions but we find in Breslich's illustrations questions on both linear and quadratic functions. The questions are simple, for example, in a linear equation $C = 2\pi r$, if r is doubled, what change in C follows--On the basis of the above mentioned authorities, the texts have been examined as to questions on changing quantities. We

1. E. R. Breslich, Measuring the Develpment of Functional Thinking in Algebra, p 99

2. N. J. Lennes, op. cit., p 62.

find a wide range, Book One having only one such question and Book Three as many as sixty-five.

Table 31: The Number of Questions on Changing Quantities in the Texts.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Questions on changing quantities	1	21	65	47	25	3	44	6	8	38

If we may assume that these questions on changing quantities may be used as one means of determining the functional aspect of an algebra text, and since we can recall that the function idea should 'not be isolated in one chapter' but should 'be manifested throughout the entire book', it seems necessary to see the distribution of these questions in the various books. Book One has its one question and Book Six its three questions in one chapter. Books Eight and Nine have their questions in two chapters and Book Three in six chapters. Since the texts do not contain a uniform number of chapters, the comparison may be better made as follows: Book Six which contains twenty-four chapters has questions about changing quantities in only one, hence four percent of its chapters contain these question, Book One with its one question and ten chapters rates ten per cent, and Book Three which also contains the greatest number of questions has the highest rating of thirty-eight per cent. In considering the number of chapters in a book, a group of supple-

mentary exercises or of extra tests was counted as another chapter.

Table 32: The Per cent of Chapters Containing Questions on Changing Quantities Compared with the Total Number of these Questions Found in each text.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Chapters	10	18	16	16	16	24	15	19	15	9
Number of Chapters Containing Questions on Changing Quantities	1	3	6	5	5	1	3	2	2	3
Per cent of Chapters Containing these Questions	10	17	38	31	31	4	20	11	13	33
Total Number of these Questions	1	21	65	47	25	3	44	6	8	38

From this diagram we can see that Book Three which has the greatest number of questions, has the highest per cent of distribution, and Books One and Six interchange their positions at the other end of the scale.

Since we are following Reeve's Objectives in the Teaching of a Formula and since we have used his List of Desirable Formulas as a basic list, it would be well to find out how the formulas used in the texts compare with the thirty-nine geometric formulas from Reeve's list. Book Ten which contains the greatest number of formulas (forty-two) also has the largest number from Reeve's list (twenty-seven), while Book Six which has the smallest number of formulas

(twenty-one) has fifteen from Reeve. Book Seven with its twenty-two formulas also has fifteen taken from Reeve's list.

Table 33: Comparison of each Book's Formulas that Coincide with the Formulas from Reeve's list.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas	32	22	38	31	34	21	22	39	25	42
Number of Formulas from Reeve's List	22	17	26	26	23	15	15	26	19	27

4. The importance of the preparation of students for college board examinations is generally conceded. Lennes says "A teacher who fails, in a marked degree, in preparing intelligent students for these examinations usually would not last long."[1] A textbook, then, should contain material in accordance with the requirements of the board. Let us see how our algebra texts compare with the following suggestions from the 1923 requirements. Of the five formulas given as types to be considered, two are geometric: #4 (Area of a trapezoid) and #27 (Volume of a sphere). These are given merely as 'types to be considered', but we may trace them through the books to see how this suggestion has been followed. The Board recommends,[2] "The meaning, use, evaluation, necessary transformation, and the derivation of such formulas from rules expressed in words." In this work "the general idea of the dependence of one

1. N. J. Lennes, op. cit., p 60

2. College Entrance Examination Board, 1923 Requirements, pp 2, 13.

variable upon another should be repeatedly emphasized." We can classify these suggestions as falling under Reeve's first (derivation), first and second (meaning). third (evaluation), fourth (transformation) and sixth (idea of dependence) objectives.

Book Five is the only text to give the derivation of formula #4 (Area of a trapezoid), but three of the texts One, Five, and Eight give this derivation for #27 (Volume of a sphere).

Table 34: The Number of Times the College Board Formula Suggestions are followed for Derivation

The Books by Number	1	2	3	4	5	6	7	8	9	10
Derivation of Formula #4	0	0	0	0	1	0	0	0	0	0
Derivation of Formula #27	1	0	0	0	1	0	0	1	0	0

All of the texts but Number Nine give the meaning of formula #4, some of the explanations being more detailed than others; and all but Four and Eight give some explanation of formula #27.

Table 35: The Number of Times the College Board Formula Suggestions are followed for Meaning

The Books by Number	1	2	3	4	5	6	7	8	9	10
Meaning of Formula #4	✓	✓	✓	✓	✓	✓	✓	✓	0	✓
Meaning of Formula #27	✓	✓	✓	0	✓	✓	✓	0	✓	✓

Formula #4 is evaluated in Book One ten times as contrasted with no evaluations in Books Six and Nine. Evalua-

tions of #27 vary from three in Book Three to none in Books One, Nine, and Ten.

Table 36: The Number of Times the College Board Formula Suggestions are followed for Evaluation

The Books by Number	1	2	3	4	5	6	7	8	9	10
Evaluations of Formula #4	10	3	9	5	6	0	1	2	0	3
Evaluations of Formula #27	0	2	3	1	2	1	1	1	0	0

These two formulas were also given as types of those to be transformed: Book Four contains ten transformations of formula #4 (Area of a trapezoid), and Books Seven, Nine, and Ten only one; Books Two, Three and Ten have formula #27 transformed once, while the remaining seven books do no change the subject of this formula.

Table 37: The Number of Times the College Board Formula Suggestions are followed for Transformation

The Books by Number	1	2	3	4	5	6	7	8	9	10
Transformations of Formula #4	5	2	3	10	3	2	1	3	1	1
Transformations of Formula #27	0	1	1	0	0	0	0	0	0	1

Since we have previously checked the 'idea of dependence by considering the number of questions on changing quantities, let us consider the number of times these questions are applied to our typical formulas. Formula #4 (Area of a trapezoid) is used in this way three times by Book Seven, twice by Book Ten, and not

once by the other eight texts. Formula #27 is used more often, the range being from eleven times in Book Seven to no times in five of the books.

Table 38: The Number of Times the College Board Formula Suggestions are followed for the Idea of Dependence.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Questions' about Formula #4	0	0	0	0	0	0	3	0	0	2
Number of Questions' about Formula #27	0	1	0	0	0	1	11	0	1	3

The geometric formulas given by the College Board as types of the material adapted to the construction and interpretation of graphs[1], are formula #11 (Area of a circle) and formula #25 (Volume of a circular cylinder), the latter being considered for the case in which an arbitrary value is assigned to one of the variables. The graph of formula #11 is required of the pupil using Book Two, twice, and not at all of those who are following Books One, Five, Seven, and Nine. Formula #25 is to be shown on the graph by the pupil using Book Ten--once--while the other books do not require its construction at all.

Table 39: The Number of Times the College Board Formula Suggestions are followed for the Graph

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Graphs to be Drawn Formula #11	0	2	1	1	0	1	0	1	0	1
Number of Graphs to be Drawn Formula #25	0	0	0	0	0	0	0	0	0	1

1. Ibid., p 2

As mentioned before, it is often required that the pupil read from the graph--for example--"From the graph of $A = \pi r^2$, find the Area of a Circle whose radius is 10." Of the six books which require the construction of the graph of formula #11 (Area of a circle), Book Two asks for four of these readings and Books Three and Four ask for none. Book Ten is the only book requiring the graph of formula #25 (Volume of a circular cylinder), and it asks for no readings. The graphs of certain formulas are sometimes given in the texts and the pupils are asked to evaluate the formula from the given graph. This method is not followed for either of the two typical formulas, so the following list of readings takes in only those from graphs constructed by the pupils.

Table 40: The Number of Times the College Board Formula Suggestions are followed for Reading from the Graphs

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Readings from Graph of Formula #11	0	4	0	0	0	1	0	3	0	1
Number of Readings from Graph of Formula #25	0	0	0	0	0	0	0	0	0	0

The summary of tables thirty-four to forty is given with the rank numbers in tables fifty-seven and fifty-eight on pages eight, eighty-one, and eighty-two.

5. Dividing the texts into two groups, the first containing the five books published between 1925 and 1928,

and the second group those published from 1929 to 1934, we find surprisingly little difference between the two groups.[1] One reason for this slight variation may be that most of the books in the second group are revisions of texts published many years ago and which in the process of revision have retained much of the old material. This comparison is made as follows: the total number of formulas exemplifying each of Reeve's six points has been previously given for each book; by combining the totals included in each of Reeve's points for the five texts in the first group, (Books One, Two, Four, Seven, and Ten) and following a similar procedure for the second group, we have totals which aid in comparing the two groups. The greatest difference is in Reeve's second point, translation of a formula into a rule of mathematics, where group one has a total of eighty-five and group two only forty-two. The unusual difference is in point six, the idea of dependence, which is stressed by the latest writers in the field. On this point, as in two others, group one rates higher, having one hundred and fifty-one questions as contrasted with one hundred and seven for the second group.

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1. See Table 67 pp 106-7

Table 41: The Comparison by means of Reeve's Six Objectives of Texts published 1925-1928 (Group One) with the Texts published 1929-1934 (Group Two)[1]

Reeve's Objectives	Translation into Formula I	Translation into Rules II	Evaluation III	Transformation IV	Graphs V
Number of Formulas					
Group One	198	85	586	152	20
Group Two	253	42	606	122	25

Reeve's Sixth Objective	Idea of dependence
Number of Questions about Changing Quantities	
Group One	151
Group Two	107

IV Comparison of the Texts.

1. Since we have discussed aims and objectives and have found Reeve's objectives in teaching the formula most representative of the whole group, and since the ten algebra texts have been examined as to the geomtric formulas employed in following these objectives, we are ready now to compare the results of the examination with the objectives. The texts have been ranked in the following way.

Table 42: The Rank Number of the Texts Depending on the Number of Formulas Used in the Texts

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas	32	22	38	31	34	21	22	39	25	42
Rank Number of the Book	5	8.5	3	6	4	10	8.5	2	7	1

1. For figures see table 67, pp 106-7

The number of formulas included in Reeve's first and second objectives or the number of formulas about which the pupil should have some idea of meaning give the texts the following rank numbers.

Table 43: The Rank Numbers of the Texts According to the Meaning Given to the Formulas.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Formulas Included in I and II (Reeve)	32	18	36	28	32	18	22	36	19	30
Rank Number of the Book	3.5	9.5	1.5	6	3.5	9.5	7	1.5	8	5

As has been mentioned before, some books include formulas for which they give no meaning, in spite of the fact that all writers agree that the pupil should have some idea of the meaning of the formula which he is using.

Table 44: The Rank Numbers of the Texts Depending on the Number of Unexplained Formulas

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Unexplained Formulas	0	4	2	3	2	3	0	3	6	12
Rank Number of the Book	1.5	8	3.5	6	3.5	6	1.5	6	9	10

The Rank Numbers resulting from the number of evaluations are as follows:

Table 45: The Rank Numbers of the Texts According to the Number of Evaluations

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Evaluations	180	90	227	159	101	76	67	145	57	90
Rank Number of the Text	2	6.5	1	3	5	8	9	4	10	6.5

The necessary transformation of a formula is one of the common objectives.

Table 46: The Rank Numbers of the Texts According to the Number of Transformations

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Transformations	21	26	29	62	24	19	3	17	33	40
Rank Number of the Text	7	5	4	1	6	8	10	9	3	2

The fourth objective, relating to graphs, is taken in its three parts; the making of tables, the construction of grapns, and the readings from the graphs.

Table 47: The Rank Numbers of the Texts According to the Number of Tables to be Constructed.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Tables of construct	0	0	12	0	6	0	0	4	1	0
Rank Number of the Text.	7.5	7.5	1	7.5	2	7.5	7.5	3	4	7.5

Table 48: The Rank Numbers of the Texts According to the Number of Graphs to be Drawn.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Graphs to Draw	1	10	11	4	2	4	2	4	4	3
Rank Number of the Text	10	2	1	4.5	8.5	4.5	8.5	4.5	4.5	7

Table 49: The Rank Numbers of the Texts According to the Number of Readings from the Graph

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Readings from Graph	7	12	18	16	25	11	2	16	19	2
Rank Number of the Text	8	6	3	4.5	1	7	9.5	4.5	2	9.5

The idea of dependence, checked by the questions on changing quantities gives Book Three its fourth Rank One.

Table 50: The Rank Numbers of the Texts Depending on the Number of Questions on Dependence.

The Books By Number	1	2	3	4	5	6	7	8	9	10
Number of Questions on Dependence	1	21	65	47	25	3	44	6	8	38
Rank Number of the Text	10	6	1	2	5	9	3	8	7	4

The percent of the chapters which contain these questions shows the spread of the idea through the book.

Table 51: The Rank Numbers of the Texts Depending on the percent of Chapters Containing Questions on Dependence.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Percent of Chapters Containing Questions on Dependence	10	17	38	31	31	4	20	11	13	33
Rank Number of the Text	9	6	1	3.5	3.5	10	5	8	7	2

There is one more point to consider here, the number of formulas in each book which are from Reeve's List of Desirable Formulas.

Table 52: Rank Numbers of the Texts Depending on the Number of Formulas Taken from Reeve's List.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Formulas from Reeve's List	22	17	26	26	23	15	15	26	19	27
Rank Number of the Text	6	8	3	3	5	9.5	9.5	3	7	1

From these eleven tables showing the rank numbers we get the following totals.

Table 53: The Summary of the Rank Numbers in Tables 42-52

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total of Eleven Rankings	69.5	73	23	47	47	89	79.	53.5	68.5	55.5

The summary of tables forty-two to fifty-three is given in table sixty-eight on page one hundred and eight These rankings place the Books in the following order:

Table 54: The Rank Order of the Texts According to Reeve's Six Objectives

Book Three	Rank 23
Book Four and Five	Rank 47
Book Eight	Rank 53.8
Book Ten	Rank 55.5
Book Nine	Rank 68.5
Book One	Rank 69.5
Book Two	Rank 73
Book Seven	Rank 79
Book Six	Rank 89

Attention should be called to the range of the total rankings. Book Three with a rank of twenty-three is twenty-four points ahead of its nearest competitors, Book Four and Five, or in other words its score is slightly less than half the score of either of

the latter. Book Six, at the other end of the distribution, does not show such a difference but it is ten points away from the next in rank.

The above ranking is one which is true and authoritative for most of the objectives discussed in Chapter Two. There is, however, one group of writers--Jalonower, Leenes, Breslich in the Seventh Yearbook and Georges-who place their emphasis so decidedly on the idea of functionality that some different scheme of ranking should be followed. In this study, we have checked functional ideas only by questions on changing quantities and by the spread of these questions through a text, the reasons for this being discussed previously. Let us separate these two ideas from the other objectives given by Reeve, in order to get from them a ranking which will in an objective way give an indication of the trend of functional ideas in a text.

Table 55: The Rank Numbers of the Texts According to the Idea of Functionality

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of questions about Changing quantities	1	21	65	47	25	3	44	6	8	38
Rank Number	10	6	1	2	5	9	3	8	7	4
Percent of Chapters containing questions about Dependence	10	17	38	31	31	4	20	11	13	33
Rank Number	9	6	1	3.5	3.5	10	5	8	7	2
Total of the two Ranking	19	12	2	5.5	8.5	19	8	16	14	6

This combination places the books in the following order.

Table 56: The Rank Order of the Texts According to the Idea of Functionality

Book Three	Rank 2
Book Four	Rank 5.5
Book Ten	Rank 6
Book Seven	Rank 8
Book Five	Rank 8.5
Book Two	Rank 12
Book Nine	Rank 14
Book Eight	Rank 16
Books One and Six	Rank 19

Comparing this result with that of the first ranking, we see that although there are changes in the center of the distribution, Books Three and Four keep their leading positions and Book Six retains its

place at the other end. Book Five which shared second place with Book Four in the first ranking drops to fifth place here, and Book Seven which previously was rated ninth is fourth on this new list.

3. Another comparison of the texts was made by means of their use of the formulas suggested by the College Board Requirements. The tables of comparison are repeated and the corresponding rank numbers follow.

Table 57: The Rank Numbers of the Texts According to their Following of the Formula Suggestions of the College Board.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Meaning of Formula #4	✓	✓	✓	✓	0	✓	✓	✓	0	✓
Rank Number	4.5	4.5	4.5	4.5	9.5	4.5	4.5	4.5	9.5	4.5
Meaning of Formula #27	0	✓	✓	0	0	✓	✓	0	✓	✓
Rank Number	8.5	3.5	3.5	8.5	8.5	3.5	3.5	8.5	3.5	3.5
Evaluation of Formula #4	10	3	9	5	6	0	1	2	0	3
Rank Number	1	5.5	2	4	3	9.5	8	7	9.5	5.5
Evaluation of Formula #27	0	2	3	1	2	1	1	1	0	0
Rank Number	9	2.5	1	5.5	2.5	5.5	5.5	5.5	9	9
Transformation of Formula #4	5	2	3	10	3	2	1	3	1	1
Rank Number	2	6.5	4	1	4	6.5	9	4	9	9
Transformation of Formula #27	0	1	1	0	0	0	0	0	0	1
Rank Number	7	2	2	7	7	7	7	7	7	2

Table 57: Rank Number, continued

The Books By Number	1	2	3	4	5	6	7	8	9	10
Derivation from rule (Formula #4)	0	0	0	0	1	0	0	0	0	0
Rank Number	6	6	6	6	1	6	6	6	6	6
Derivation from rule (Formula #27)	1	0	0	0	1	0	0	1	0	0
Rank Number	2	7	7	7	2	7	7	2	7	7
Number of Questions on Dependence (#4)	0	0	0	0	0	0	3	0	0	2
Rank Number	6.5	6.5	6.5	6.5	6.5	6.5	1	6.5	6.5	2
Number of Questions on Dependence (#27)	0	1	0	0	0	1	11	0	1	3
Rank Number	8	4	8	8	8	4	1	8	4	2
Number of Graphs Formula(#11)	0	2	1	1	0	1	0	1	0	1
Rank Number	8.5	1	4	4	8.5	4	8.5	4	8.5	4
Number of Graphs (Formula #25)	0	0	0	0	0	0	0	0	0	1
Rank Number	6	6	6	6	6	6	6	6	6	1
Number of Readings (Graph of #11)	0	4	0	0	0	1	0	3	0	1
Rank Number	7.5	1	7.5	7.5	7.5	3.5	7.5	2	7.5	3.5
Number of Readings (Graph of #25)	0	0	0	0	0	0	0	0	0	0
Rank Number	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5

Table 58: The Summary of Table 57

The Books by Number	1	2	3	4	5	6	7	8	9	10
Totals of the above Fourteen Rankings	82	61.5	67.5	81	79.5	79	80	76.5	98.5	64.5

These totals rank the texts in the following order

Table 59: The Rank Order of the Texts According to Table 57

Book Two	Rank 61.5
Book Ten	Rank 64.5
Book Three	Rank 67.5
Book Eight	Rank 76.5
Book Six	Rank 79
Book Five	Rank 79.5
Book Seven	Rank 80
Book Four	Rank 81
Book One	Rank 82
Book Nine	Rank 98.5

This ranking presents a different picture, Book Three which held the first rank twice is changed to third and Book Six which held the last rank both preceeding times is now changed to fifth place. Note should also be made of the apparent disregard of the four formulas suggested by the College Board Requirements.

4. Since the later writers in the field strongly emphasize the idea of functional thinking which Reeve includes in his list as his sixth objective, a final com-

parison may be made by grouping the Ranking from Revere with the second ranking which is based on the idea of dependence. This gives extra weight to Reeve's sixth point which aids in giving it the emphasis suggested by such writers as Lennes, Jablonwer, and Georges.

Table 60: The Combination of the Numbers from Reeve's Objectives and from the Idea of Functionality

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Rank from Reeve	69.5	73	23	47	47	89	79	53.5	68.5	55.5
Total Rank Dependence	19	12	2	5.5	8.5	19	8	16	14	6
Total of the two Ranks	88.5	85	25	52.5	55.5	108	87	69.5	82.5	61.5

This combination gives the texts the following rank order.

Table 61: The Rank Order of the Texts According to Table 60

Book Three	Rank 25
Book Four	Rank 52.5
Book Five	Rank 55.5
Book Ten	Rank 61.5
Book Eight	Rank 69.5
Book Nine	Rank 82.5
Book Two	Rank 85
Book Seven	Rank 87
Book One	Rank 88.5
Book Six	Rank 108

Book Three is decidedly in first place, again having a rank less than half of the following rank and Book Six definitely keeps its last position by being nineteen and one-half points away from its predecessor. There are slight changes in the center of the distribution

V Conclusion

The marked differences in the content of the ten first year algebra texts examined is seen by even a glance at the data which have been presented in this study. Before drawing any conclusions, however, attention must be called to the fact that only geometric formulas and only those geometric formulas which are translations of general rules have been included. Although there is reason to believe that there would also be differences if all types of formulas had been considered, still we have no objective evidence to substantiate this belief and therefore must limit our conclusions accordingly.

The differences in the algebra texts form the most outstanding conclusion. As far as geometric formulas are concerned, the texts vary in the formulas which they use, in the number of formulas which they use, and also in their treatment of the formulas. Two of the texts explain all of the formulas which they give, and one of the texts leaves one fourth of its formulas without any clue as to their meaning.[1] The range of the distribution of the number of evaluations, from fifty-seven to two hundred and twenty-seven shows the difference clearly.[2] After a study of the literature for aims and objectives it was found possible to adopt one set of objectives as representative of the writers

1. See Table 15, p 58

2. See Table 16, p 59

with the exception of the group emphasizing functionality. There does not seem to be the same division in the texts. These differences are more remarkable if we recall the objectives given in the prefaces of the texts. Nine of the texts, all but Book Three, state in the preface that the material is based on one or more of the following: The aims of the National Committee, the requirements of the College Board, and Objectives which are similar to those given by Reeve. From this uniformity, we get the following result: Book Three the only text not mentioning any of the above aims has rank one with a lead of more than half the rank of the next text.[1] The range of the whole distribution is from twenty-three to eighty-nine; the range not including Book Three, and thus grouping those texts which have the same expressed aims, is from forty-seven to eighty-nine.[1] Let us try another; the second ranking, based on the idea of dependence will indicate the texts which emphasize funcional thinking.[2] Three of the texts Five, Eight and Nine mention this point in their prefaces, Nine being the one emphasizing it most. The second ranking shows Book Five in fifth, Book Nine in seventh, and Book Eight in eighth place.[2] Again the unexpected.

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1. See Table 54, p 78

2. See Table 56, p 79

Since only two of the texts examined in this study were published as recently as 1934, it may be assumed that the material for all of them was prepared before the 1933 appointment by the College Entrance Examination Board of the Commission on Examinations in Mathematics. Therefore the texts should be compard ontheir following of the 1923 requirements of the board. The first ranking includes the recommendations of the College Board, but a separate check was made on the particular formulas suggested as types, which resulted in the third ranking. In this third ranking we find agrement among the texts, agreement in ignoring the suggested formulas. The agreement was not universal in every respect, the range of the rank numbers being from sixty-one and five-tenths to ninety-eight and five-tenths.[1] Four of the texts, Books Two, Four, Seven, and Eight mention the College Board Requirements as one of their objectives and should therefore have the preferable ranks. Book Two is consistent, it is based on the College Board and has rank one, Book Eight is fourth, Book Seven seventh, and Book Four eighth on the list.[1] Since the Commission on Examinations in Mathematics has recently published its report containing the new requirements which emphasize funcional thinking, the third rank order will mean little for future use.

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1. See Table 59. p 82

The second ranking, which is based on the idea of dependence is the one which follows the new thought in the field. This ranking is limited to the two ideas on functional thinking which permit of objective treatment: questions about dependence or changing quantities and the spread of these questions throughout the text. The first ranking includes the idea of dependence tho this idea comprises only two-elevenths of the basic criteria. In a previous chapter mention was made of the fact that the later writers in their training in functional thinking will require some of the old ideas--that is, meaning, evaluation, and transformation. The graph as a representation of a function is an idea advocated by both groups. Hence the new idea appears to be similar to the old but with a decided change in emphasis. For these reasons it seemed advisable to combine the first and second ranking, the result being one which does not go all the way with the new thought, but emphasizes functional thinking more than in the original first ranking. A text book which ranks high on this combination would be a good transition text. Book Three is again far ahead of the others in rank and Book Six is still holding tenth place, the range being from twenty-five to one hundred and eight.[1]

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1. See Table 61, p 83

Attention should be called again to the fact that Book Three has a decided lead in the important rankings and is third in the ranking according to the old requirements of the College Board. Book Four shares the second place with Book Five in the first ranking but holds it alone in the other two and Book Six is definitely place tenth except in the ranking for the College Board formulas.

From these conclusions based on data relating to geometric formulas, we arrive at the following results:

If one wishes to find out which of the authors most carefully followed the old requirements of the College Board, reference may be made to the third ranking.[1]

If a text which will follow the objectives of most of the writers on mathematical aims is required, the best choice will be one of the upper group in the first ranking.[2]

If a text which will aid in teaching functional thinking is required, the preference will be one of the upper group in the second ranking.[3]

If a text which will merely emphasize functional thinking is required, the best choice will be from the combination ranking.[4]

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1. See Table 59, p 82
2. See Table 54, p 78
3. See Table 56, p 79
4. See Table 61, p 83

VI Bibliography

1. Breslich, Ernest R., "Understanding and Mechanical Performance in Algebra," The Mathematics Teacher, XXV, Feb. 1935, pp 57-65.

2. Breslich, Ernest R., The Technique of Teaching Secondary School Mathematics, Chicago: University of Chicago Press, 1930, pp 1-239

3. Breslich, Ernest R., "Measuring the Development of Functional Thinking in Algebra," The Seventh Yearbook of the National Council of Teachers of Mathematics, New York City: Bureau of Publications Teachers College, Columbia University, 1932, pp 93-118

4. Breslich, Ernest R., "Secondary School Mathematics and the Changing Curriculum," The Mathematics Teacher, XXVI, Oct. 1933, p 328.

5. Blackhurst, James Herbert, Principles and Methods of Junior High School Mathematics, Boston, Ginn and Company, 1930, p 82

6. College Entrance Examination Board, Definition of the Requirements in Elementary Algebra, Document 107, May 1923, pp 1, 13.

7. College Entrance Examination Board, Report of the Commission on Examinations in Mathematics, March 1935, pp 159-160

8. Georges, J. S., "Functional Thinking as an Objective of Mathematical Education", School Science and Mathematics, XXIX, May June 1929 pp 508-15, 601-08.

9. Jablonower, Joseph, "Recent and Present Tendencies in the Teaching of Algebra in the High Schools," The Seventh Yearbook of the National Council of Teachers of Mathematics, New York City, Bureau of Publications Teachers College Columbia University, 1932, pp 1-18.

10 Lennes, N. J., "The Function Concept in Elementary Algebra," The Seventh Yearbook of the National Council of Teachers of Mathematics, Bureau of Publications Teachers College Columbia University, 1932, pp 52-73.

11. Lide, Edwin S., "Mathematics in the Changing Curriculum of the High School." National Education Association Proceedings, 1923, pp 491-492.

12. Longley, W. R., "The Function Concept in Algebra," The Mathematics Teacher, XXVI, Jan. 1933, p 6.

13. Maxwell, C. R., The Selection of Textbooks, Boston, Houghton Mifflin Company, 1921, pp 62-77, 124,

14. The National Committee on Mathematical Requirements, The Reorganization of Mathematics in Secondary Education, Mathematical Association of America, 1923, pp 6-16, 32-33, 62-66, 92-105

15. Reeve, W. D., "Objectives in the Teaching of Mathematics," The Mathematics Teacher, XVIII, Nov. 1925, pp 385-401.

16. Schorling, Raleigh, A Tentative List of Objectives in the Teaching of Junior High School Mathematics, Ann Arbor Michigan, George Wahr 1925, pp 3-4, 9, 70-74, 87-96, 105-106, 119.

17. Schultze, Arthur, The Teaching of Mathematics in Secondary Schools, Boston, The Macmillan Company 1912, pp 15-29.

18. Smith, David Eugene, and Reeve, William David, "Objectives in the Teaching of Junior High School Mathematics," The Second Yearbook of the National Council of Teachers of Mathematics, Bureau of Publications, Teachers College Columbia University, 1927, p 198.

19. Smith, David Eugene, and Reeve, William David, The Teaching of Junior High School Mathematics, Boston, Ginn and Company 1927, pp 204-08.

20. Whitcraft, L. H., "Some of the Influences of the Requirements and Examinations of the College Entrance Examination Board on Mathematics in the Secondary Schools of the United States," Abstract Teachers College Record, XXV, Dec. 1933, pp 234-5.

VII Appendix

The First Year Algebra Texts Examined in this Study with the Numbers used to Designate them.

1. Barber, Harry C., "Everyday Algebra for the Ninth School Year," Houghton Mifflin, Boston, 1925
2. Durell, Fletcher and Arnold, E.E., "A First Book in Algebra," Charles E. Merrill Company, New York, 1927.
3. Edgerton, Edward I. and Carpenter, Perry A., "Elementary Algebra", Allyn and Bacon, Boston 1929
4. Engelhardt, Fred and Haertter, Leonard D., "First Course in Algebra", John C. Winston Company, Philadelphia, 1928.
5. Hart, Walter W., "Progressive First Algebra", D. C. Heath and Company, Boston, 1934.
6. Hawkes, Herbert E., Luby, William A., And Touton, Frank C., "First Year Algebra", Ginn and Company Boston, 1934.
7. Longley, Wm. Raymond and Marsh, Harry Brooks, Algebra Book One", The Macmillan Company, New York 1927.
8. Milne, William J. and Downey, Walter F., "First Course in Algebra", American Book Company, Boston, 1933.
9. Schorling, Raleigh and Clark, John R, "Modern Algebra First Course", World Book Company, New York, 1929.

10. Stone, John C., "The New Mathematics: First Year Algebra", Benjamin H. Sanborn and Company, Boston, 1928.

List of Geometric Formulas taken from Reeve and from the ten texts.

1.	$A = lw$	Area of a rectangle
2.	$A = bh$	Area of a parallelogram
3.	$A = \frac{1}{2} bh$	Area of a triangle
4.	$A = h/2 (b+b')$	Area of a trapezoid
5.	$P = 2 (l+w)$	Perimeter of a rectangle
6.	$P = 4 s$	Perimeter of a square
7.	$P = a+b+c$	Perimeter of a triangle
8.	$P = 3s$	Perimeter of an equilateral triangle
9.	$C = 2\pi r$	Circumference of a circle
10.	$C = \pi d$	Circumference of a circle
11.	$A = \pi r^2$	Area of a circle
12.	$A = \pi d^2/4$	Area of a circle
13.	$A = \frac{1}{2} ap$	Area of a regular polygon
14.	$S = 2 lw+2 lh+2hw$	Total area of a rectangular solid
15.	$S = 6 e^2$	Total area of a cube
16.	$L = 2\pi r h$	Lateral area of a circular cylinder
17.	$S = 2\pi rh+2\pi r^2$	Total area of a circular cylinder
18.	$L = \pi rl$	Lateral area of a circular cone
19.	$L = Cl/2$	Lateral area of a circular cone
20.	$S = \pi rl-\pi r^2$	Total area of a circular cone
21.	$S = 4\pi r^2$	Area of a sphere
22.	$S = \pi d^2$	Area of a sphere
23.	$V = lwh$	Volume of a rectangular solid
24.	$V = e^3$	Volume of a cube

25.	$V = \pi r^2 h$	Volume of a circular cylinder
26.	$V = Bh$	Volume of a circular cylinder
27.	$V = 4/3\pi r^3$	Volume of a sphere
28.	$V = 1/6\pi d^3$	Volume of a sphere
29.	$A = \pi(R^2 - r^2)$	Area of a ring
30.	$c^2 = a^2 + b^2$	Pythagorean relation
31.	$c = 360/n$	Central angle of a circle
32.	$a + b + c = 180$	Sum of the angles of a triangle
33.	$d = \sqrt{a^2 + b^2 + c^2}$	Diagonal of a cube
34.	$A = n\pi r^2/360$	Area of a sector of a circle
35.	$l = n\pi d/360$	Length of an arc of a circle
36.	$h = a/2\sqrt{3}$	Altitude of an equilateral triangle
37.	$A = a^2/4\sqrt{3}$	Area of an equilateral triangle
38.	$d = a\sqrt{2}$	Diagonal of a square
39.	$E = 1.1547R$	Cube inscribed in a sphere
40.	$S = \pi r$	Length of a semicircle
41.	$C = 2\sqrt{\pi A}$	Circumference of a circle in terms of the area.
42.	$s = ac/360$	Length of an arc.
43.	$P = 2s + b$	Perimeter of an isosceles triangle
44.	$P = 2c + a + b$	Perimeter of an isosceles trapezoid
45.	$P = ns$	Perimeter of a regular polygon
46.	$S = 4(l + w + h)$	Sum of the lengths of the edges of a rectangular solid
47.	$S = 12e$	Sum of the lengths of the edges of a cube

48.	$4m^2 = 2(a^2 + b^2) - c^2$	Median of a triangle
49.	$D = n/2\ (n-3)$	Number of diagonals of a polygon having n sides
50.	$P = 5c$	Perimeter of a pentagon
51.	$A = c^2/4\pi$	Area of a circle
52.	$A = \frac{1}{2}\ d^2$	Area of a rhombus
53.	$K = \sqrt{s(s-a)\ (s-b)\ (s-c)}$	Area of a triangle
54.	$A = 2h\ (l+w) + lw$	Lateral area of the sides and one base of a rectangular solid
55.	$A = 2l\ (r+s) + r^2 + s^2$	Area of the frustum of a square pyramid
56.	$S = \frac{1}{2}\ pl$	Surface of a pyramid
57.	$S = 2e^2 + 4\ eh$	Surface of a prism with a square base
58.	$L = \pi sr_1 + \pi sr_2$	Area of the curved surface of the frustume of a cone.
59.	$A = b\sqrt{d^2 - b^2}$	Area of a rectangle in terms of its diagonal and base.
60.	$A = \pi(R+2r)(R-2r)$	Area left when four circular holes are cut from a large circular plate
61.	$A = s^2$	Area of a square
62.	$A = (36\pi V^2)^{1/3}$	Area of the surface of a sphere
63.	$A = \pi r^2 E/180$	Area of a triangle on a sphere
64.	$A = .7854\ d^2$	Area of a circle
65.	$V = \pi R^2 h - \pi r^2 h$	Volume of a hollow cylinder
66.	$V = 1/3\pi r^2 h$	Volume of a cone
67.	$V = 1/3\pi h\ (l^2 - h^2)$	Volume of a cone

68.	$V = 1/3\ as^2$	Volume of a square pyramid
69.	$V = \pi r^2/2\ (h+h')$	
70.	$V = Bh$	Volume of a prism
71.	$V = h/6\ (a+b+4m)$	Volume of a prismatoid
72.	$V = 1/3\ Bh$	Volume of a pyramid
73.	$V = 1/4\pi d^2h$	Volume of a circular cylinder
74.	$V = 1/3\ lwh$	Volume of a pyramid
75.	$V = sh^2$	Volume of a prism with a square base.
76.	$V = \frac{1}{3}\ Bh$	
77.	$V = 1/3\pi h\ (r^2 + r^2 + rr_1)$	Volume of the frustum of a cone
78.	$V = \frac{h}{3}\ (b_1 + b_2 + \sqrt{b_1 b_2})$	Volume of the frustum of a pyramid
79.	$V = A^{\frac{3}{2}}/6\pi$	Volume of a sphere
80.	$V = 1/3\ Bh$	Volume of a cone
81.	$A_1 + A_2 = 180$	Two supplementary angles
82.	$x+y = 90$	Two complementary angles
83.	$A = (n-2)\ 180$	Sum of the angles in any polygon
84.	$A = \frac{n-2}{n}\ 180$	Each angle of an equiangular polygon

Table 1: The total Number of Formulas Found in each Text.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of formulas found in each book	32	22	38	31	34	21	22	39	25	42

Table 62: The Number of Unexplained Formulas Found in the Ten Texts

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Unexplained formulas	0	4	2	3	2	3	0	3	6	12

Table 63: The Number of Different Formulas used in Reeve's Third, Fourth, Fifth, and Sixth Points

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Different formulas used in Reeve's III (Evaluation)	30	18	34	26	24	14	16	34	18	27
Number of different formulas used in Reeve's IV (Transformation)	9	13	18	16	13	10	3	10	13	24
Number of different formulas used in Reeve's V (Graphs)	1	5	10	4	7	3	2	4	5	4
Number of different formulas used in Reeve's VI (Dependence)	1	6	17	14	10	6	8	3	9	11

Table 64: The Number of Formulas in each Book Found only in that Text

Books by Number	1	2	3	4	5	6	7	8	9	10
Number of formulas found in only one book	2	2	5	2	3	2	3	6	1	7

Table 33: The Comparison of each Book's Formulas that Coincide with the Formulas from Reeve's list.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of Formulas	32	22	38	31	34	21	22	39	25	42
Number of Formulas from Reeve's List	22	17	26	26	23	15	15	26	19	27

Table 2: The Number of Formulas Common to the Various Books together with the Specific Numbers of the Formulas Found in the Texts.

The table reads: Of the eighty-four formulas, six are found in all ten books. They are #3, 4, 11, 25, 27, 30. (as numbered on pages 42-46)

Total Number of Books	10	9	8	7	6	5	4	3	2	1	0
Total Number of Formulas found in the different Books.	6	6	3	7	4	4	3	4	9	33	5
The Specific Numbers of the formulas found in the different books (For reference see List of Formulas pages 42-46)	#3 4 11 25 27 30	#1 5 6 9 23 61	#17 21 66	#2 10 16 24 32 70 72	#7 29 36 38	#14 15 37 54	#8 28 53	#20 26 71 80	#12 33 42 43 47 56 81 82 84	#13 18 19 40 41 44 45 46 48 49 50 51 52 55 57 58 59 60 62 63 64 65 67 68 69 73 74 75 76 77 78 79 83	#22 31 34 35 39

Table 3: Comparison of the Most Widely Used Formulas as Found in this Study with Those Found in Schorling's Inventory of the Content of Seven Series of Junior High School Mathematics Texts and with Those Listed in Book Four.

The Numbers of the Formulas Found in Six or more Books	The Formulas Found in Schorling's Study which correspond with Column I	The Formulas Given in the list in Book Four which correspond with Column I
#3	✓	✓
4	✓	
11	✓	✓
25	✓	✓
27	✓	
30		
1	✓	✓
5		
6		
9	✓	
23	✓	✓
61	✓	✓
17		✓
21	✓	
66	✓	
2	✓	✓
10	✓	✓
16	✓	✓
24		✓
32		
70	✓	
72	✓	
7		
29		
36		
38		
	Also #26, #18, interest, percentage, distance	Also #15 interest, distance

Table 65: The Specific Numbers of the Unexplained Formulas Found in each Text

The Books by Number	1	2	3	4	5	6	7	8	9	10
The Numbers of the Unexplained Numbers	0	#3 20 55 69	#17 42	#6 27 28	#17 71	#38 42 71	0	#17 48 64	#4 9 11 61 70 72	#2 9 10 16 23 56 66 71 73 74 75 76

Table 66: The Number of Times Formulas are Used in Each of Reeve's Six Objectives

I Translation of a Rule into a Formula--Combination of Tables 4-11.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Formulas given slight explanation	4	5	17	5	1	11	9	1	3	12
Formulas given detailed explanation	5	3	5	15	7	4	6	4	3	2
Formula written from given rule as review	3	0	8	0	2	0	0	1	1	2
originally	8	0	8	2	18	0	1	11	2	1
Formula derived from previous knowledge	4	6	3	3	3	3	0	3	9	2
Formula given after rule is repeated from arithmetic	1	0	1	0	0	4	5	11	3	6
Formula derived from another formula	1	0	3	0	0	2	2	1	3	2
Formula derived from figure and explanation	0	3	0	3	3	1	1	6	3	5
Formula written as review	11	12	33	32	14	0	0	20	17	16

Table 66 continued

II Translation of a Formula into a Rule--Combination of Tables 12-14

The Books by Number	1	2	3	4	5	6	7	8	9	10
Formula given, rule given	0	1	0	0	0	0	0	0	0	0
Formula given, pupil writes it as a rule originally	30	9	7	0	2	0	0	6	5	3
as review	25	11	15	5	1	0	0	3	3	1
Directions when to use formula	0	0	0	0	0	0	0	0	11	0

III Evaluation of a Formula--Table 16

The Books by Number	1	2	3	4	5	6	7	8	9	10
Evaluation for given subject	131	65	171	135	54	51	31	119	43	64
Evaluation for another letter	49	10	18	21	42	2	25	16	1	15
Evaluation of formula after subject has been changed	0	15	38	3	5	23	11	10	13	11
Total	180	90	227	159	101	76	67	145	57	90

IV Transformation of a Formula--Combination of Tables 18-24

The Books by Number	1	2	3	4	5	6	7	8	9	10
Type $2x = 6$	3	15	9	19	6	0	0	4	26	17
$x+5 = 8$	4	1	4	13	4	5	3	3	1	4
$x-4 = 7$	0	0	0	0	0	0	0	0	0	0
$\frac{1}{2}x = 9$	10	3	6	23	10	2	0	5	5	9
Radical	4	7	10	7	4	12	0	5	1	8
Total	21	26	29	62	24	19	3	17	33	40

Table 66 continued

V Graph of a Formula--Combination of Tables 25, 28, 29

The Books by Number	1	2	3	4	5	6	7	8	9	10
Construct table	0	0	12	0	6	0	0	4	1	0
Construct graph	1	10	11	4	2	4	2	4	4	3
Read values from graph	7	12	18	16	25	11	2	16	19	2

VI Idea of Dependence--Table 31

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of questions about changing quantities	1	21	65	47	25	3	44	6	8	38

Table 32: The Percent of Chapters Containing Questions on Changing Quantities Compared with the Total Number of these Questions Found in each Text

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of Chapters	10	18	16	16	16	24	15	19	15	9
Number of Chapters containing questions about dependence	1	3	6	5	5	1	3	2	2	3
Percent of Chapters containing questions on dependence	10	17	38	31	31	4	20	11	13	33
Total Number of these questions	1	21	65	47	25	3	44	6	8	38

Table 67: of which Table 41 is the summary.

Detailed Comparison by means of Reeve's Six Objectives of the Earlier with the more Recent Texts.

Group I Texts published 1925-1928			
Number of formulas found in Books	Translation of rule into formula	Translation of formula into rule	Evaluation of a formula
#1	37	55	180
#2	29	21	90
#4	60	5	159
#7	24	0	67
#10	48	4	90
Total	198	85	586

Group II Texts published 1929-1934			
Number of formulas found in Books	Translation of rule into formula	Translation of formula into rule	Evaluation of a formula
#3	78	22	227
#5	48	3	101
#6	25	0	76
#8	58	9	145
#9	44	8	57
Total	253	42	606

Table 67 continued

Group I Texts published 1925-1928			
Number of formulas found in Books	Transformation of a formula	Graph of a formula	Questions about de - pendence
#1	21	1	1
#2	26	10	21
#4	62	4	47
#7	3	2	44
#10	40	3	38
Total	152	20	151

Group II Texts published 1929-1934			
Number of formulas found in Books	Transformation of a formula	Graph of a formula	Questions about de- pendence
#3	29	11	65
#5	24	2	25
#6	19	4	3
#8	17	4	6
#9	33	4	8
Total	122	25	107

Table 68: Ranking of the Texts According to Reeve's Objectives--Combination of Tables 42-53

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Number of formulas	32	22	38	31	34	21	22	39	25	42
Rank Number	5	8.5	3	6	4	10	8.5	2	7	1
Number of formulas included in Reeve's I and II	32	18	36	28	32	18	22	36	19	30
Rank Number	3.5	9.5	1.5	6	3.5	9.5	7	1.5	8	5
Number of unexplained formulas	0	4	2	3	2	3	0	3	6	12
Rank Number	1.5	8	3.5	6	3.5	6	1.5	6	9	10
Number of evaluations	180	90	227	159	101	76	67	145	57	90
Rank Number	2	6.5	1	3	5	8	9	4	10	6.5
Number of transformations	21	26	29	62	24	19	3	17	33	40
Rank Number	7	5	4	1	6	8	10	9	3	2
Number of tables to construct	0	0	12	0	6	0	0	4	1	0
Rank Number	7.5	7.5	1.	7.5	2	7.5	7.5	3	4	7.5
Number of graphs to draw	1	10	11	4	2	4	2	4	4	3
Rank Number	10	2	1	4.5	8.5	4.5	8.5	4.5	4.5	7

Table 68 continued

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of readings from graph	7	12	18	16	25	11	2	16	19	2
Rank Number	8	6	3	4.5	1	7	9.5	4.5	3	9.5
Number of questions about dependence	1	21	65	47	25	3	44	6	8	38
Rank Number	10	6	1	2	5	9	3	8	7	4
Percent of chapters containing questions about dependence	10	17	38	31	31	5	20	11	13	33
Rank Number	9	6	1	3.5	3.5	10	5	8	7	2
Number of formulas from Reeve's list	22	17	26	26	23	15	15	26	19	27
Rank Number	6	8	3	3	5	9.5	9.5	3	7	1
Total Ranking	69.5	73	23	47	47	89	79	53.5	68.5	55.5

Table 54: Rank Order of the Texts According to Reeve's Six Objectives

Book Three	Rank 23
Book Four and Five	Rank 47
Book Eight	Rank 53.5
Book Ten	Rank 55.5
Book Nine	Rank 68.5
Book One	Rank 69.5
Book Two	Rank 73
Book Seven	Rank 79
Book Six	Rank 89

Table 55: Ranking of the Texts for Idea of Functionality

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of questions about changing quantities	1	21	65	47	25	3	44	6	8	38
Rank Number	10	6	1	2	5	9	3	8	7	4
Percent of chapers containing questions about dependence	10	17	38	31	31	4	20	11	13	33
Rank Number	9	6	1	3.5	3.5	10	5	8	7	2
Total Ranking	19	12	2	5.5	8.5	19	8	16	14	6

Table 56: Rank Order of the Texts--Idea of Functionality

Book Three	Rank 2
Book Four	Rank 5.5
Book Ten	Rank 6
Book Seven	Rank 8
Book Five	Rank 8.5
Book Two	Rank 12
Book Nine	Rank 14
Book Eight	Rank 16
Books One and Six	Rank 19

Tables 57-58: Ranking of the Texts According to Their Following of the Formula Suggestions of the College Board.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Meaning of formula #4	✓	✓	✓	✓	0	✓	✓	✓	0	✓
Rank Number	4.5	4.5	4.5	4.5	9.5	4.5	4.5	4.5	9.5	4.5
Meaning of formula #27	0	✓	✓	0	0	✓	✓	0	✓	✓
Rank Number	8.5	3.5	3.5	8.5	8.5	3.5	3.5	8.5	3.5	3.5
Evaluation of formula #4	10	3	9	5	6	0	1	2	0	3
Rank Number	1	5.5	2	4	3	9.5	8	7	9.5	5.5
Evaluation of formula #27	0	2	3	1	2	1	1	1	0	0
Rank Number	9	2.5	1	5.5	2.5	5.5	5.5	5.5	9	9
Transformation of formula #4	5	2	3	10	3	2	1	3	1	1
Rank Number	2	6.5	4	1	4	6.5	9	4	9	9
Transformation of formula #27	0	1	1	0	0	0	0	0	0	1
Rank Number	7	2	2	7	7	7	7	7	7	2

Table 57-58 continued

The Books by Number	1	2	3	4	5	6	7	8	9	10
Derivation from rule (formula #4)	0	0	0	0	1	0	0	0	0	0
Rank Number	6	6	6	6	1	6	6	6	6	6
Derivation from rule (formula #27)	1	0	0	0	1	0	0	1	0	0
Rank Number	2	7	7	7	2	7	7	2	7	7
Number of questions about dependence (formula #4)	0	0	0	0	0	0	3	0	0	2
Rank Number	6.5	6.5	6.5	6.5	6.5	6.5	1	6.5	6.5	2
Number of questions about dependence (formula #27)	0	1	0	0	0	1	11	0	1	3
Rank Number	8	4	8	8	8	4	1	8	4	2
Number of graphs (formula #11)	0	2	1	1	0	1	0	1	0	1
Rank Number	8.5	1	4	4	8.5	4	8.5	4	8.5	4
Number of graphs (formula #25)	0	0	0	0	0	0	0	0	0	1
Rank Number	6	6	6	6	6	6	6	6	6	1

Table 57-58 continued

The Books by Number	1	2	3	4	5	6	7	8	9	10
Number of readings from graph (formula #11)	0	4	0	0	0	1	0	3	0	1
Rank Number	7.5	1	7.5	7.5	7.5	3.5	7.5	2	7.5	3.5
Number of readings (graph of #25)	0	0	0	0	0	0	0	0	0	0
Rank Number	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5
Total Rank Number	82	61.5	67.5	81	79.5	79	80	76.5	98.5	64.5

Table 59: Rank Order or the Texts According to Tables 57-58

Book Two	Rank 61.5
Book Ten	Rank 64.5
Book Three	Rank 67.5
Book Eight	Rank 76.5
Book Six	Rank 79
Book Five	Rank 79.5
Book Seven	Rank 80
Book Four	Rank 81
Book One	Rank 82
Book Nine	Rank 98.5

Table 60: Combination of Ranking from Reeve's Objectives and from the Idea of Functionality.

The Books by Number	1	2	3	4	5	6	7	8	9	10
Total Rank from Reeve	69.5	73	23	47	47	89	79	53.5	68.5	55.5
Total Rank Idea of Depencence	19	12	2	5.3	8.5	19	8	16	14	6
Total of the above two ranks	88.5	85	25	52.5	55.5	108	87	69.5	82.5	61.5

Table 61: Rank Order of the Texts from Combination Ranking in Table 60

Book Three	Rank 25
Book Four	Rank 52.5
Book Five	Rank 55.5
Book Ten	Rank 61.5
Book Eight	Rank 69.5
Book Nine	Rank 82.5
Book Two	Rank 85
Book Seven	Rank 87
Book One	Rank 88.5
Book Six	Rank 108

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